



MIPT Internal Science School

Quantum transport in Dirac semimetal based Josephson junctions

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- **Summary**

Majorana fermion



Ettore Majorana

1928: Dirac equation (spin-1/2 fermion)

$$i\hbar \frac{d\psi}{dt} = c\alpha \cdot p + mc^2\beta \quad \alpha = \begin{pmatrix} \sigma & 0 \\ 0 & -\sigma \end{pmatrix}, \quad \beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$$

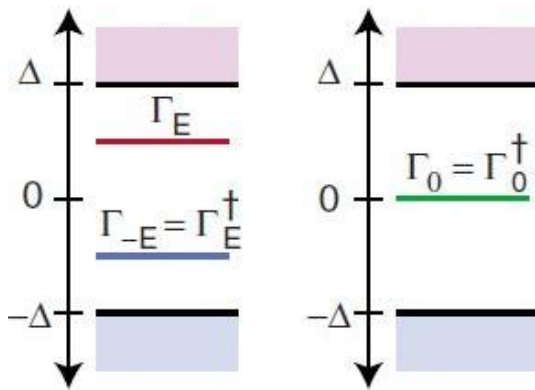
$\psi^\dagger \neq \psi \Rightarrow$ charged, particle \neq antiparticle

1937: When α is real, β is complex, then $\psi^\dagger = \psi$

\rightarrow **Majorana fermion**, **zero charge, particle = antiparticle**

In condensed matter physics, Majorana quasiparticle excitations can appear like **Majorana zero energy bound state** or **Majorana zero mode**.

Superconductor:

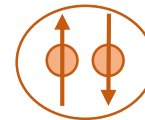


Conditions of superconductor for Majorana:

□ S-wave pairing:

$$d = ac_\uparrow^\dagger + bc_\downarrow$$

$$d^\dagger = a^\dagger c_\uparrow + b^\dagger c_\downarrow$$

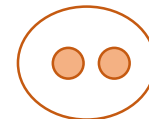


$\Rightarrow d \neq d^\dagger$

□ P-wave pairing :

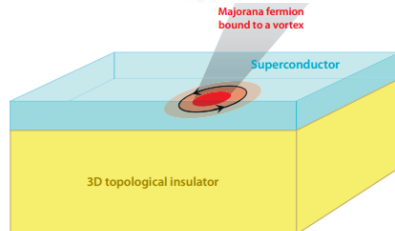
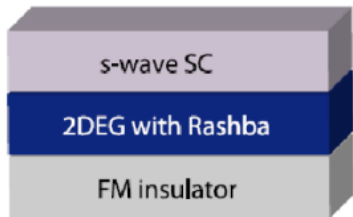
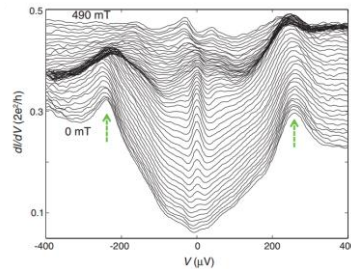
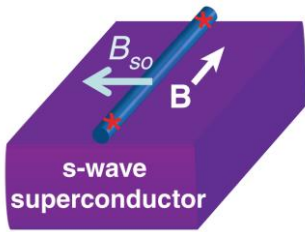
$$d = ac_\sigma^\dagger + a^\dagger c_\sigma$$

$$d^\dagger = a^\dagger c_\sigma + ac_\sigma^\dagger$$

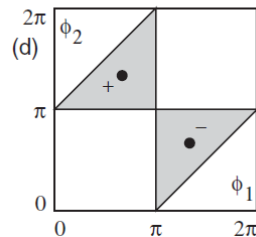
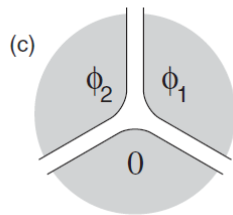
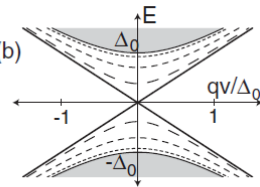
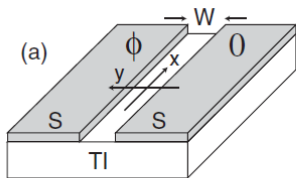


$\Rightarrow d = d^\dagger$

Majorana zero mode



TMZM in the vortex core



Fu-Kane proposal

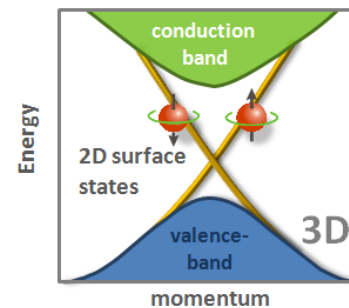
Possible systems for Majorana zero modes:

- p-wave superconductor
- Fractional quantum Hall system ($5/2$)

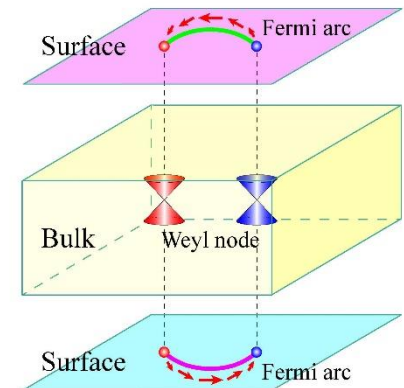
Artificial p-wave superconductivity :

- Semiconductor nanowire / superconductor
- 2D electron gas / superconductor
- **Topological insulator / superconductor (2008, Fu and Kane)**
- **Topological semimetal / superconductor**

Topological insulator



Topological semimetal



Ref:

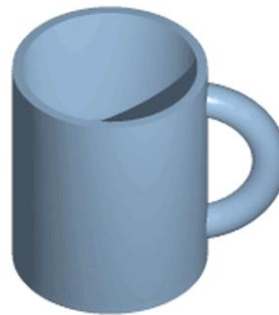
V. Mourik *et al.*, *Science* **336**, 1003-1007 (2012). C.W.J. Beenakker, *Annual Review of Condensed Matter Physics* **4**, 113-136 (2013).
C. Liu *et al.*, *National Science Review* **1**, 36-37 (2014). Phys Rev Lett **100**, 096407 (2008).

Topology

What is topology:

Topology, branch of mathematics. Topology studies properties of spaces that are invariant under any continuous deformation. It is sometimes called "rubber-sheet geometry" because the objects can be stretched and contracted like rubber, but cannot be broken.

Two objects are considered equivalent if they can be continuously deformed into one another through such motions in space as bending, twisting, stretching, and shrinking while disallowing tearing apart or gluing together parts.



A continuous transformation can turn a coffee mug into a donut. Hence a coffee mug is topologically equivalent to a donut.

Topology

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genus:



$g=0$



$g=1$



$g=2$



$g=3$

Classification of materials

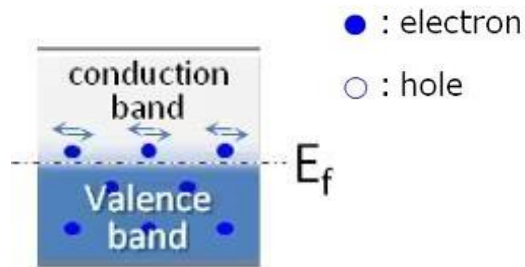
metal



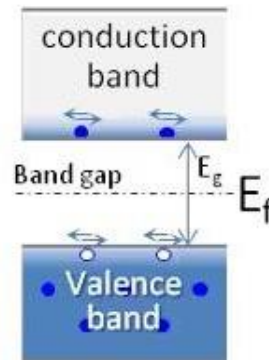
semiconductor



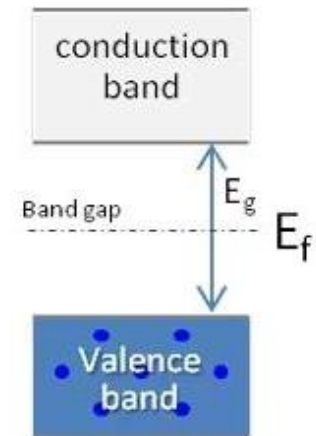
insulator



metal



semiconductor



insulator

Topological Insulator

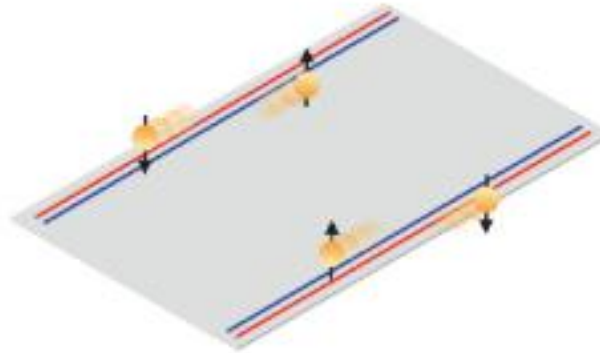
➤ Topological insulator:



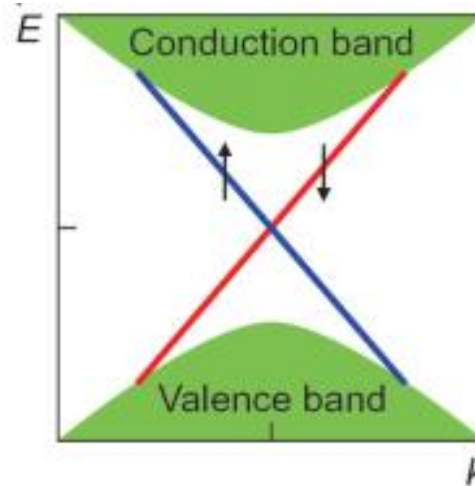
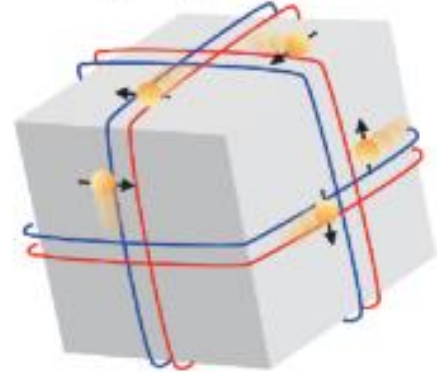
Properties:

- Bulk behaves as an insulator, while the surface behaves as a conductor.
- The surface states are topologically protected, and prevented from back-scattering
- Spin-momentum locking
- High mobility, low power dissipation

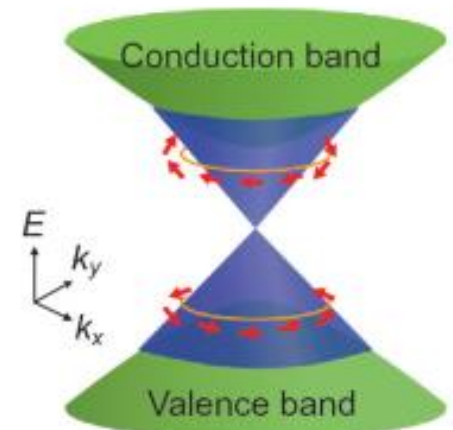
2D topological insulator



3D topological insulator



Dirac band

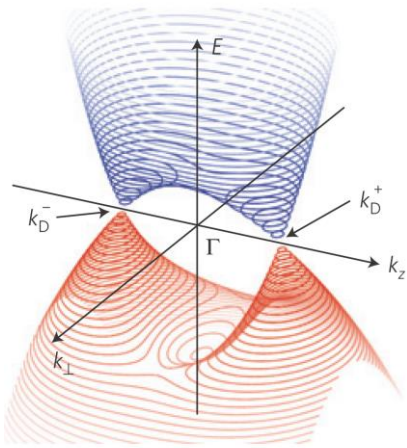


Dirac cone

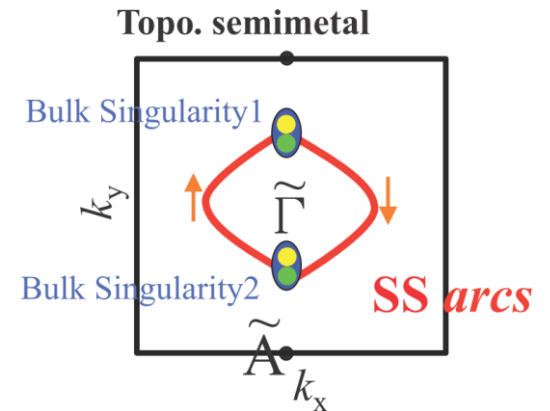
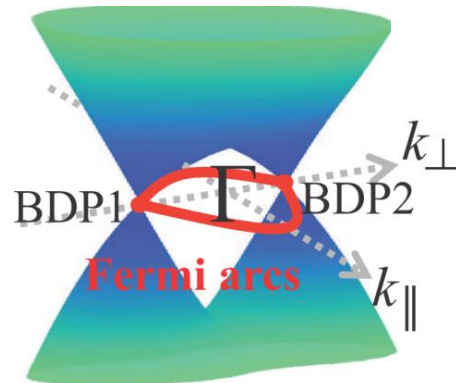
Topological Dirac semimetal

➤ Topological Dirac semimetal

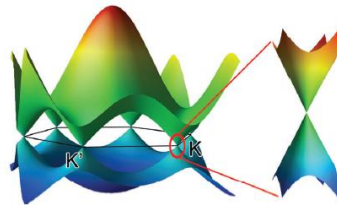
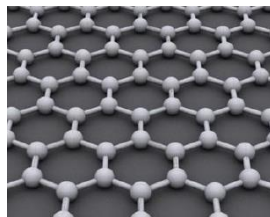
3D bulk Dirac band



Fermi-arc surface states



Also be called
“3D graphene”



graphene

Topological Dirac semimetal

Dirac Hamiltonian:

$$H = \begin{pmatrix} v\vec{\sigma} \cdot \vec{k} & 0 \\ 0 & -v\vec{\sigma} \cdot \vec{k} \end{pmatrix}$$

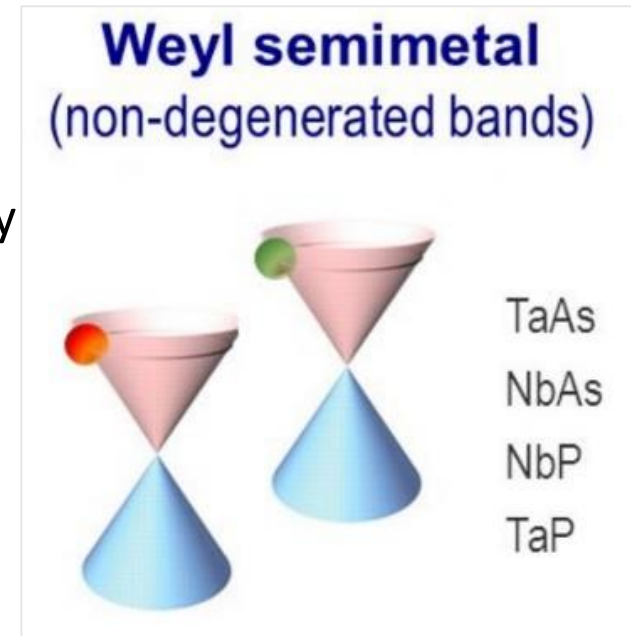
Weyl Hamiltonian:

$$H = \pm v\vec{\sigma} \cdot \vec{k}$$

One Dirac fermion can be viewed as two copies of Weyl fermions with opposite chirality.

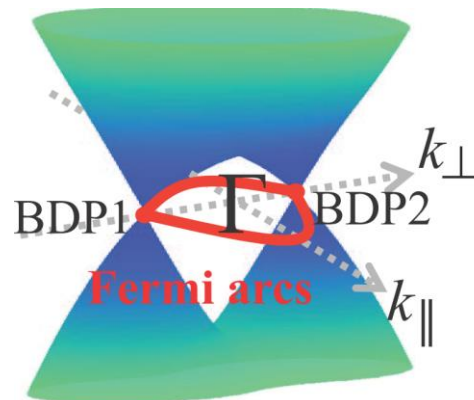
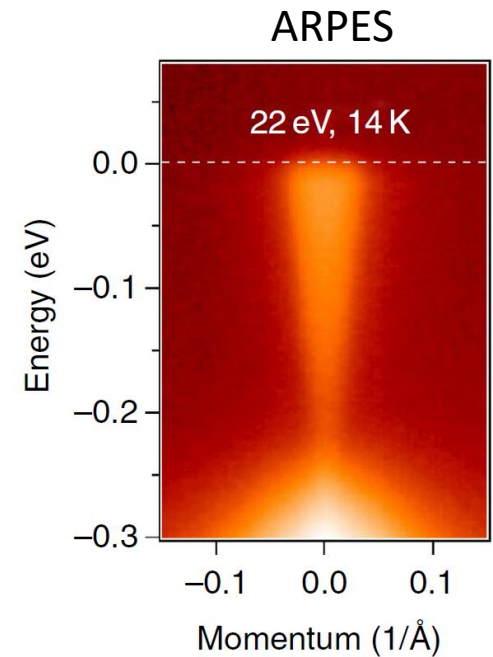
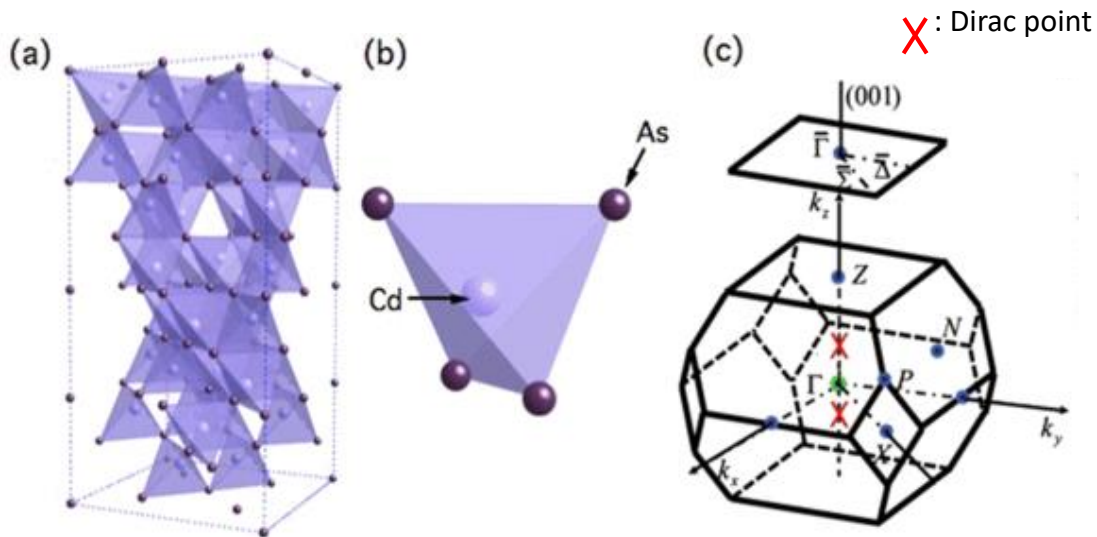


Breaking
time-reversal symmetry
or inversion symmetry



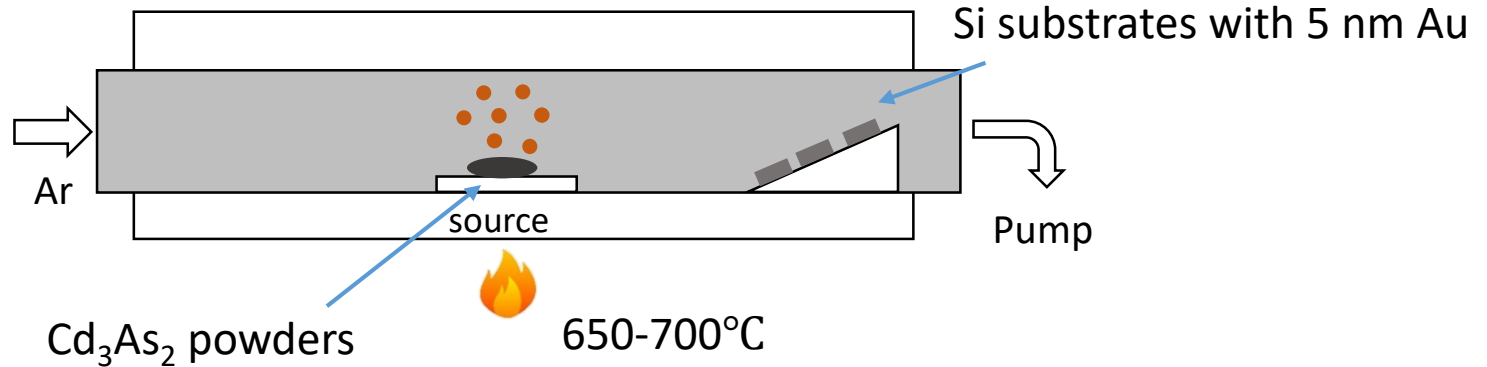
Topological Dirac semimetal: Cd_3As_2

Cd_3As_2 structure:



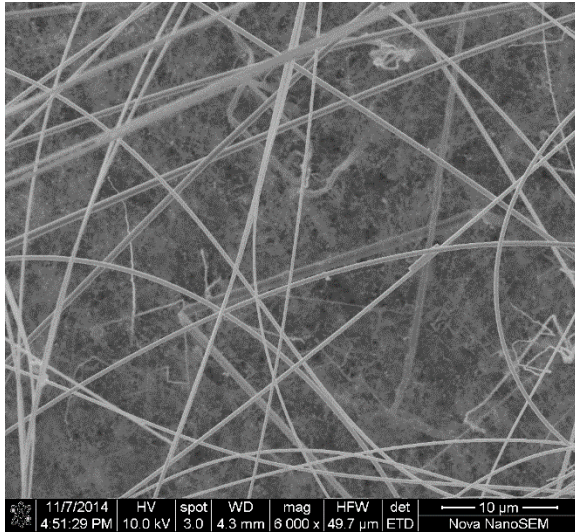
1. Synthesis and characterization of Dirac semimetal Cd_3As_2 nanostructures

CVD method:



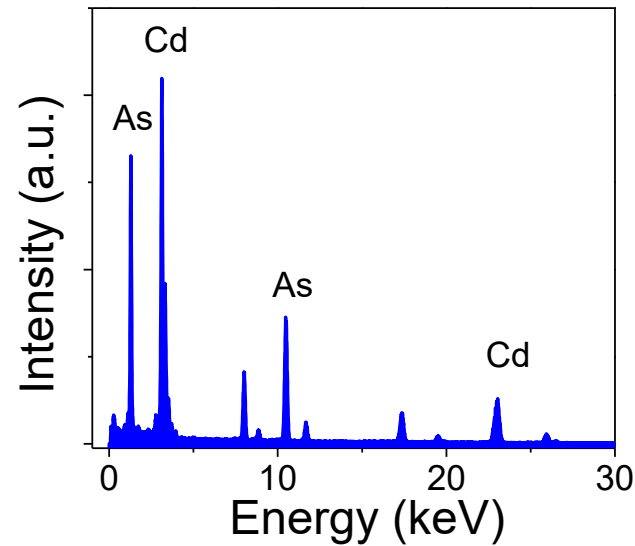
Characterization of Cd₃As₂ nanostructures

SEM

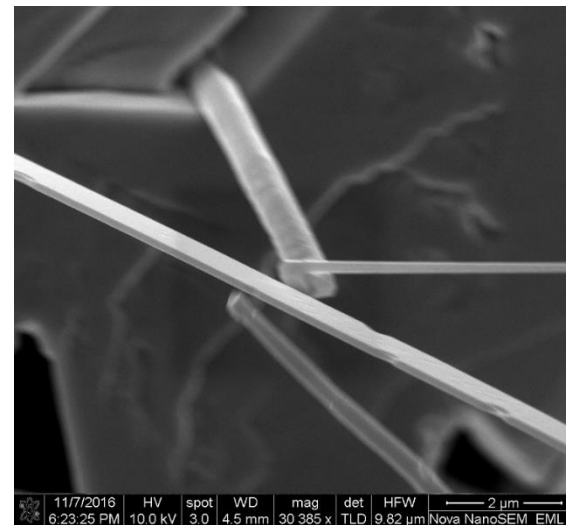
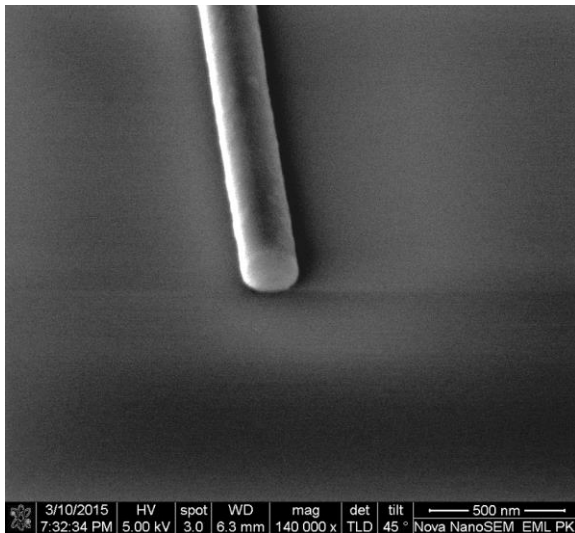


length:
~100 µm
diameter:
50-500 nm

EDS



$$\frac{\text{Cd}}{\text{As}} = \frac{58.3}{41.7} \approx \frac{3}{2}$$



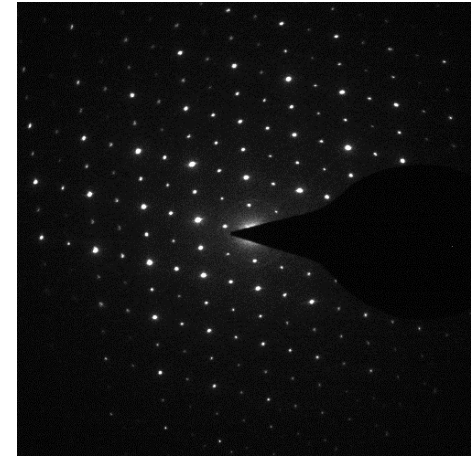
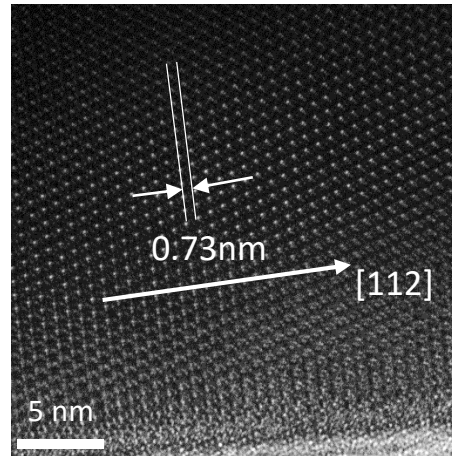
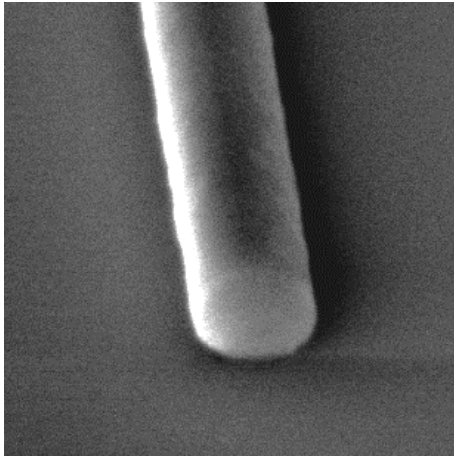
Characterization of Cd_3As_2 nanostructures

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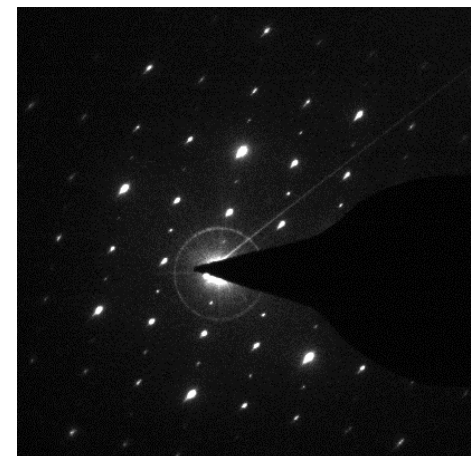
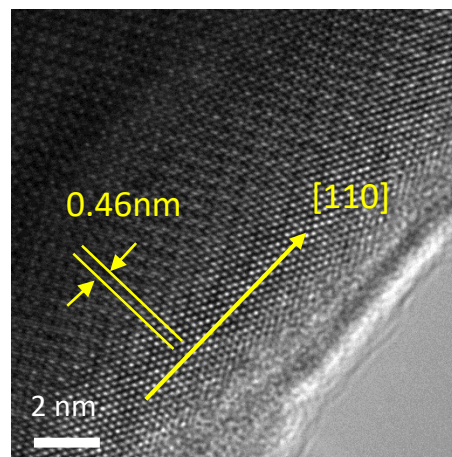
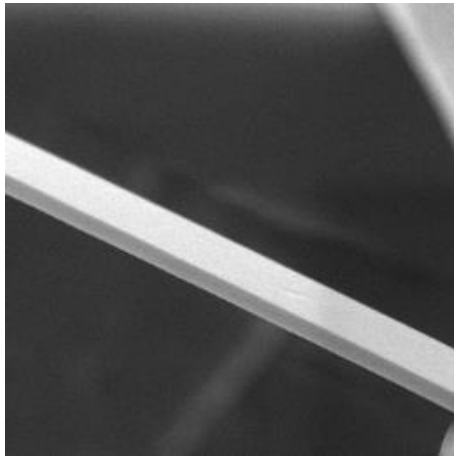
TEM

Selected area electron diffraction

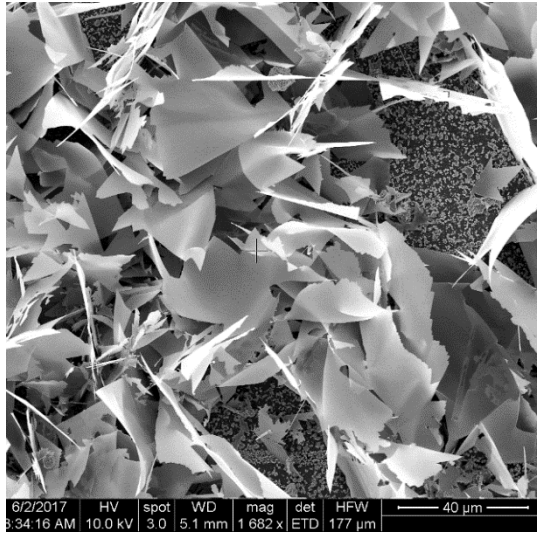
[112] axis



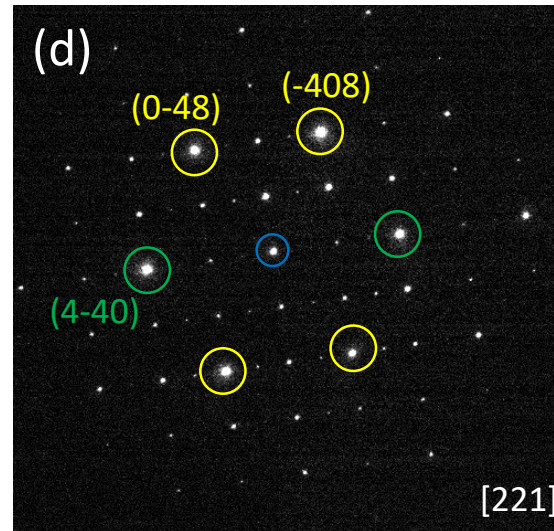
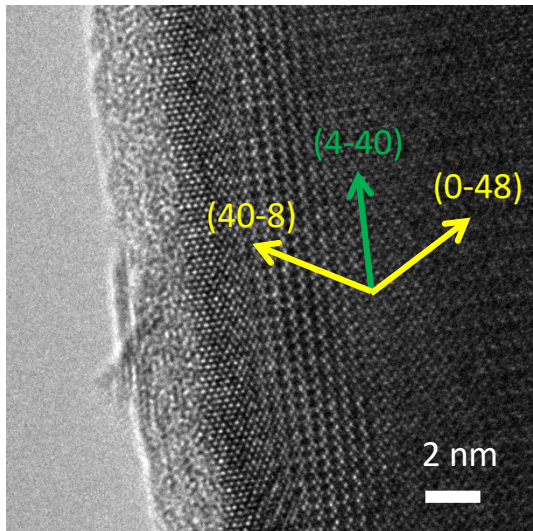
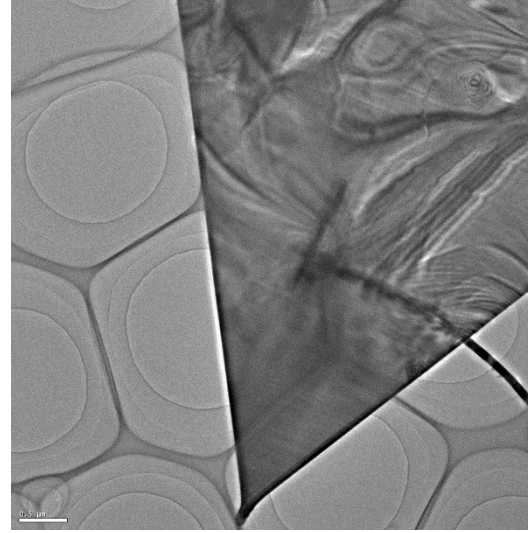
[110] axis



Characterization of Cd_3As_2 nanostructures



length:
~x0 μm
thickness:
~x0 nm

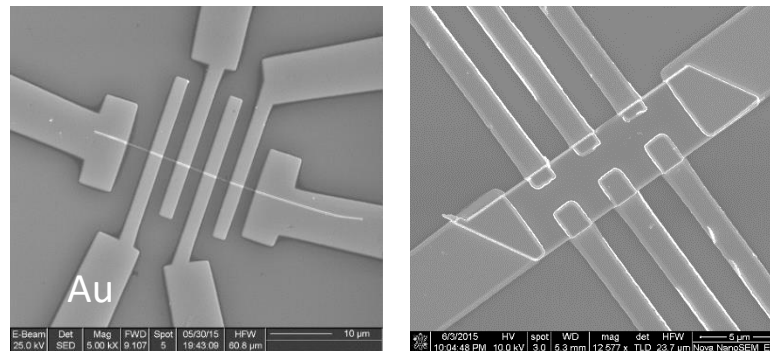


surface:
(112) surface;
edge direction:
[1-10] axis

2. Transport properties of Cd_3As_2

- I. Chiral anomaly effect in Cd_3As_2 nanowires and nanoplates
- II. Aharonov-Bohm oscillations in Cd_3As_2 nanowires
- III. Two carrier transport in Cd_3As_2 nanoplates

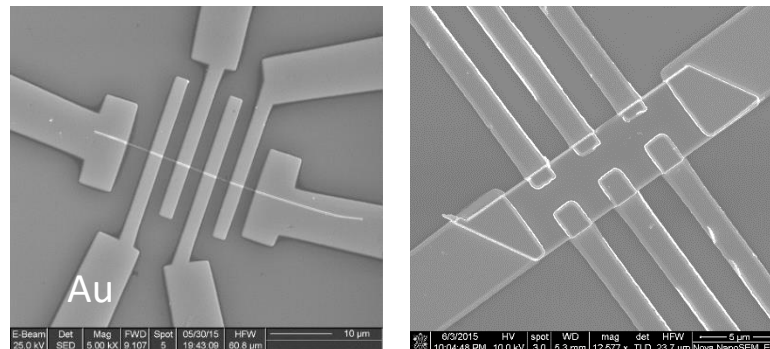
Cd_3As_2 device



2. Transport properties of Cd_3As_2

- I. Chiral anomaly effect in Cd_3As_2 nanowires and nanoplates
- II. Aharonov-Bohm oscillations in Cd_3As_2 nanowires
- III. Two carrier transport in Cd_3As_2 nanoplates

Cd_3As_2 device



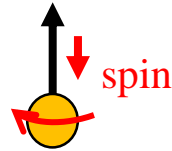
Chiral anomaly effect

Weyl fermion: chirality

$$\chi = \frac{\vec{\sigma} \cdot \vec{p}}{|\vec{p}|}$$

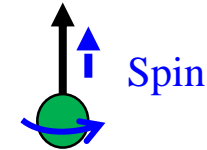


Momentum

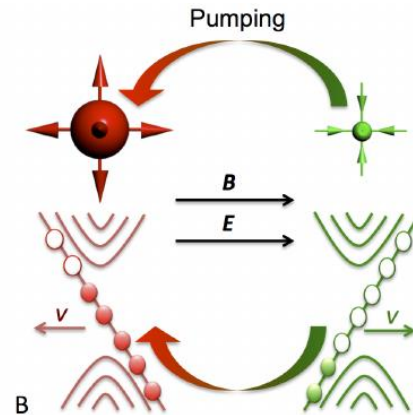
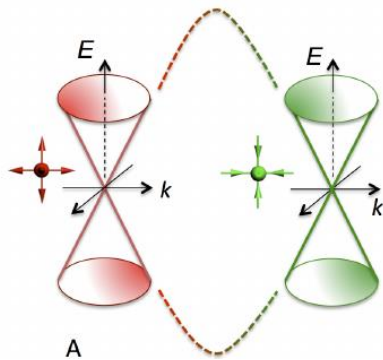


Left handed

Momentum



Right handed



$$\partial_\mu j_\chi^\mu = -\chi \frac{e^3}{4\pi^2 \hbar^2} \mathbf{E} \cdot \mathbf{B}$$

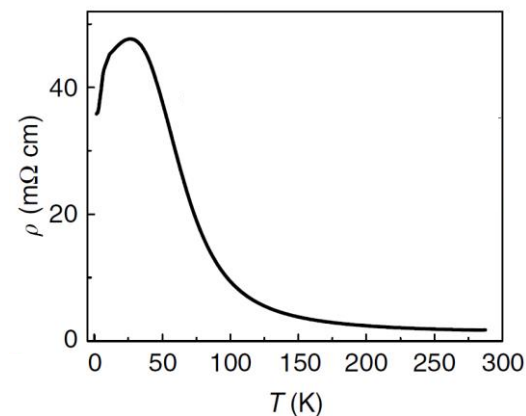
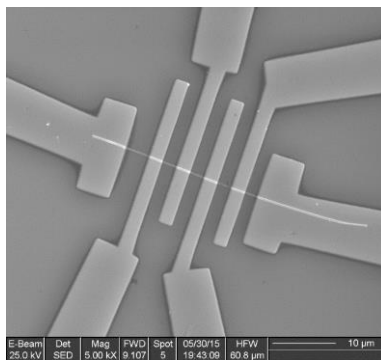
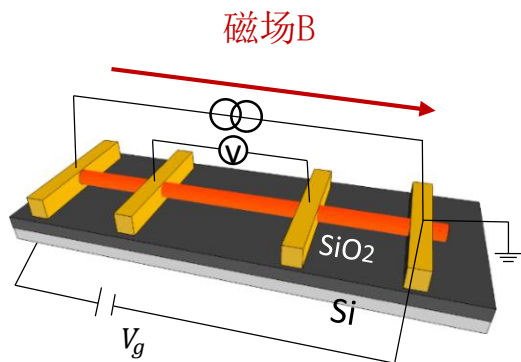
$$J_C = \left(\frac{e}{h}\right)^2 (\mu_L - \mu_R) B$$

In the transport measurements:

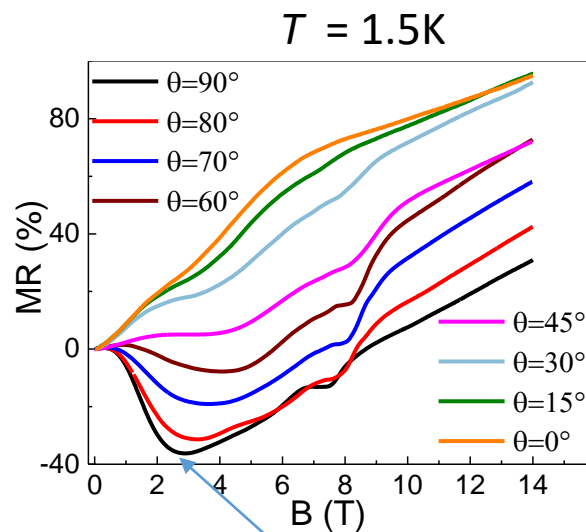
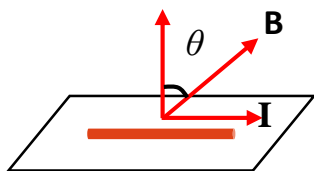


negative magnetoresistance (MR)

Negative magnetoresistance



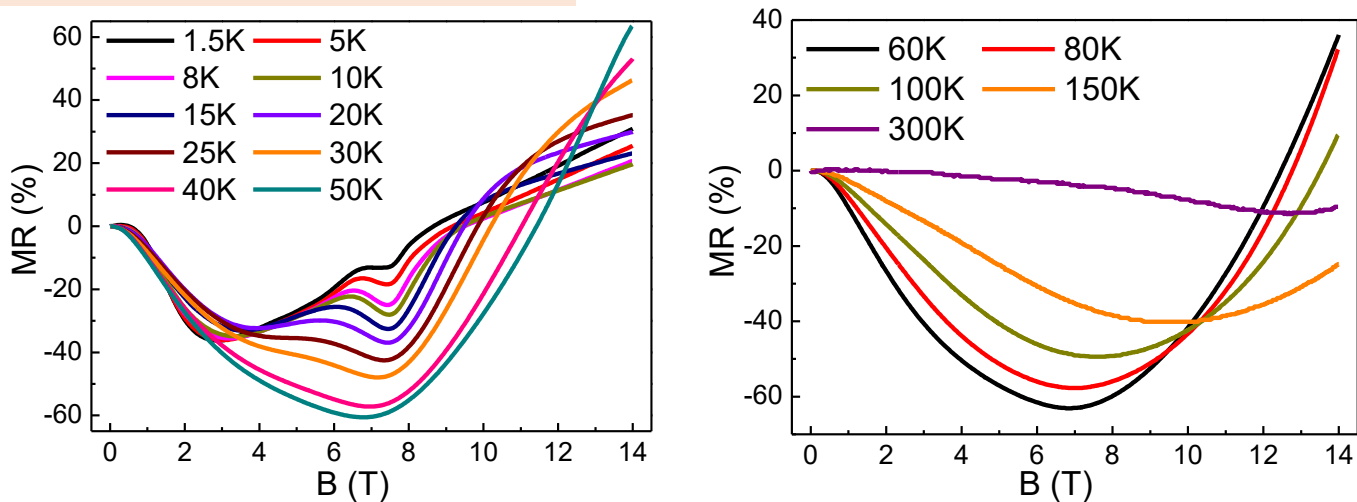
Angular-dependence of MR:



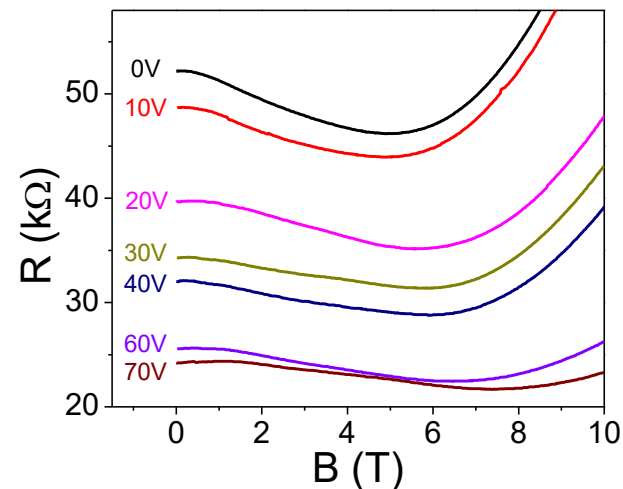
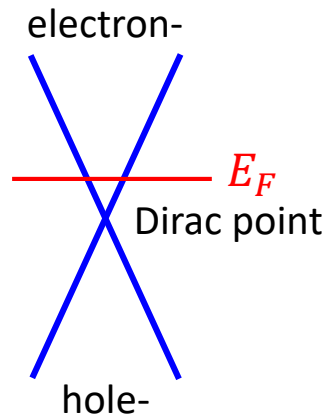
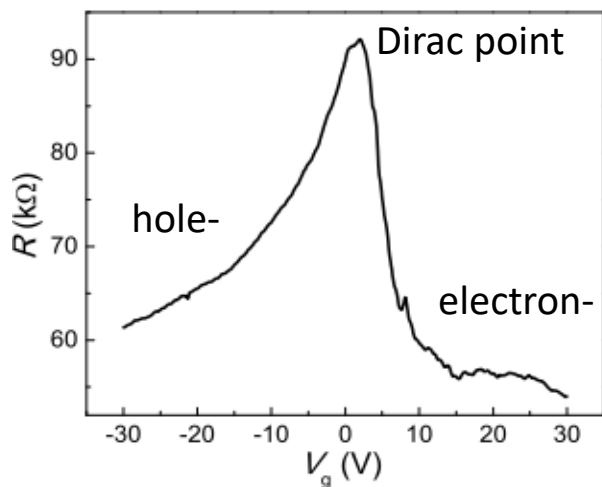
negative MR

Negative magnetoresistance

Temperature-dependence of negative MR :



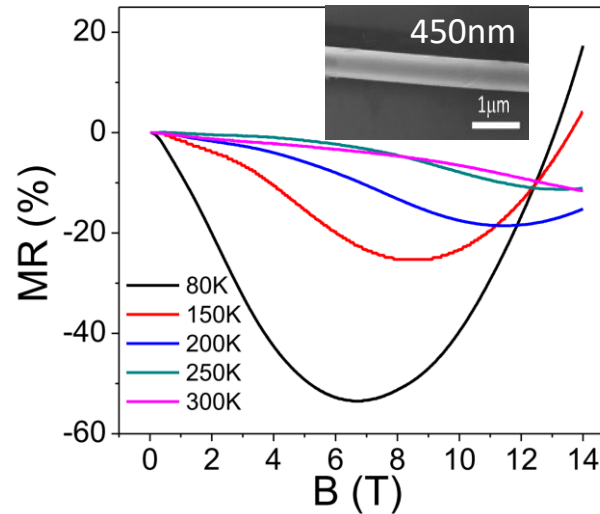
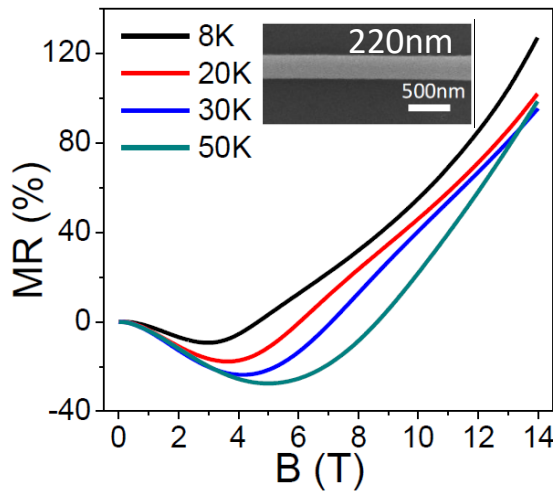
Gate tunable negative MR :



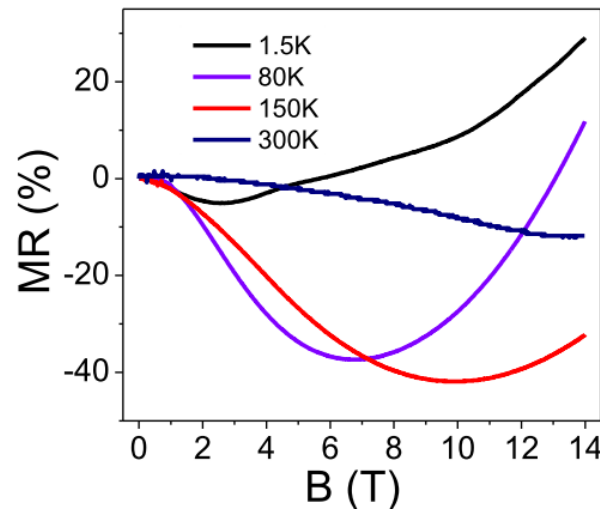
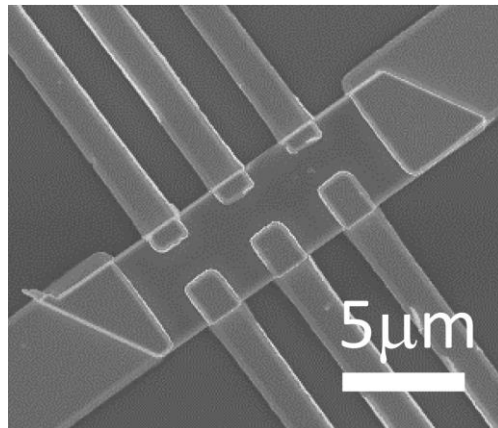
Low carrier concentration $n \sim 10^{17}/cm^3$ ($V_g = 0$ V)

Negative magnetoresistance

nanowire



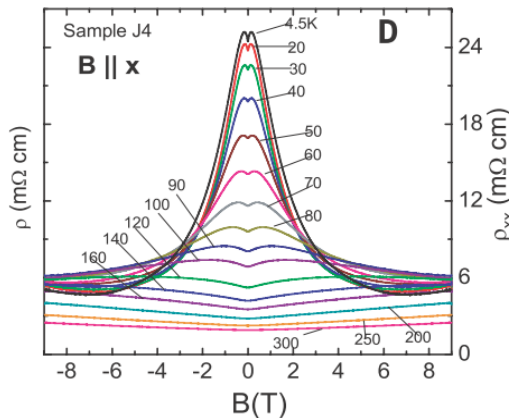
nanoplate



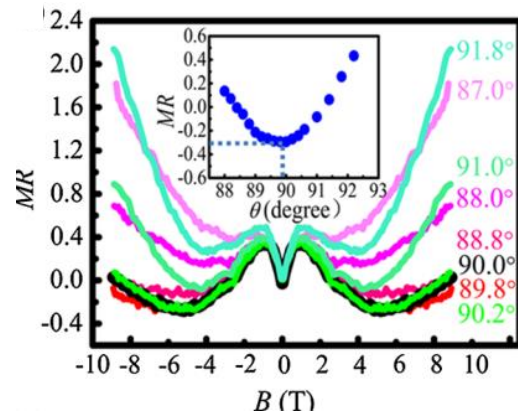
The negative MR can be observed in all nanowires devices with different diameters, as well as in nanoplate devices, indicating that the negative MR is independent of the geometry or size of the sample, and is an intrinsic property of the material.

Negative magnetoresistance

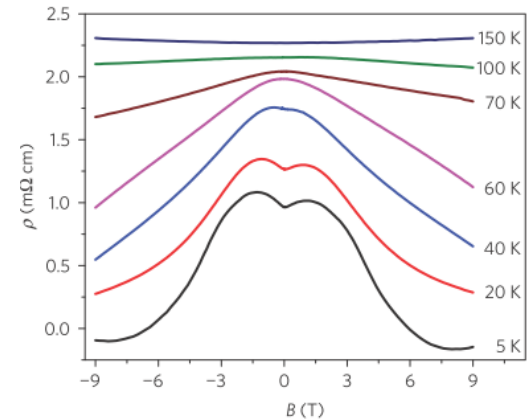
Negative MR in other materials:



Na₃Bi *Science*. **350**.413 (2015)
Princeton, Ong group



TaAs *Phys. Rev. X* **5**, 031023 (2015)
Chinese Academy of Sciences
Genfu Chen Group



ZrTe₅ *Nat. Phys.* **12**, 550 (2016)
Brookhaven National Lab, Valla Group

Our work (highly cited paper):

1



Giant negative magnetoresistance induced by the chiral anomaly in individual Cd₃As₂ nanowires

Li, CZ; Wang, LX; (...); Yu, DP

Dec 2015 | *NATURE COMMUNICATIONS* 6

Dirac electronic materials beyond graphene and topological insulators have recently attracted considerable attention. **Cd₃As₂** is a Dirac semimetal with linear dispersion along all three momentum directions and can be viewed as a three-dimensional analogue of graphene. By breaking of either time-reversal symmetry or spatial inversion symmetry, the Dirac semimetal is believed to transform into a W ... [Show more](#)

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342

Citations

33

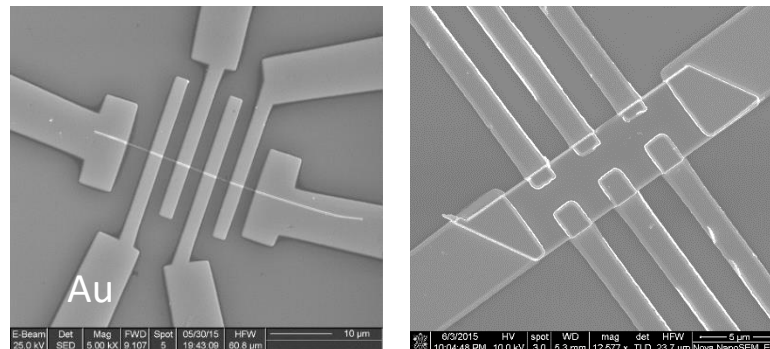
References

[Related records](#)

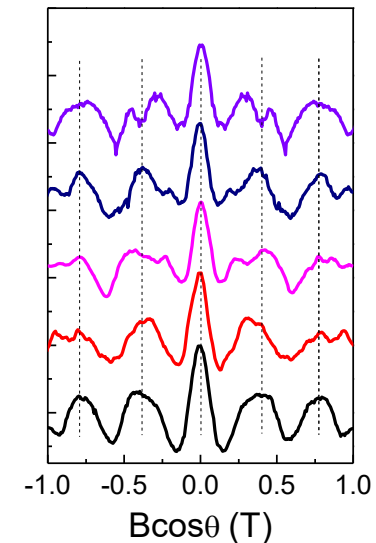
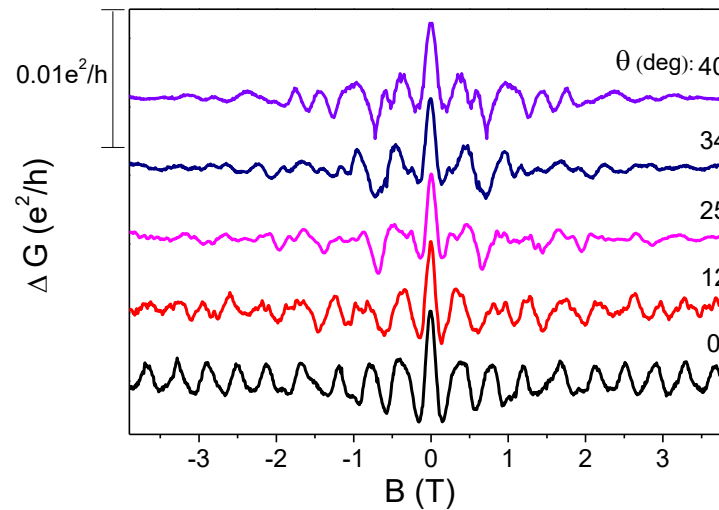
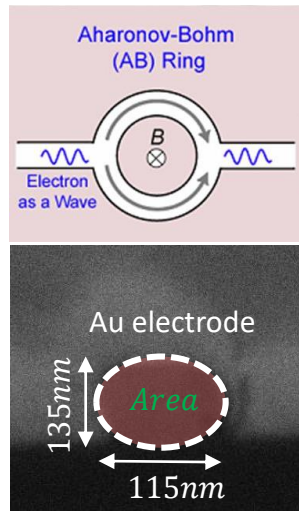
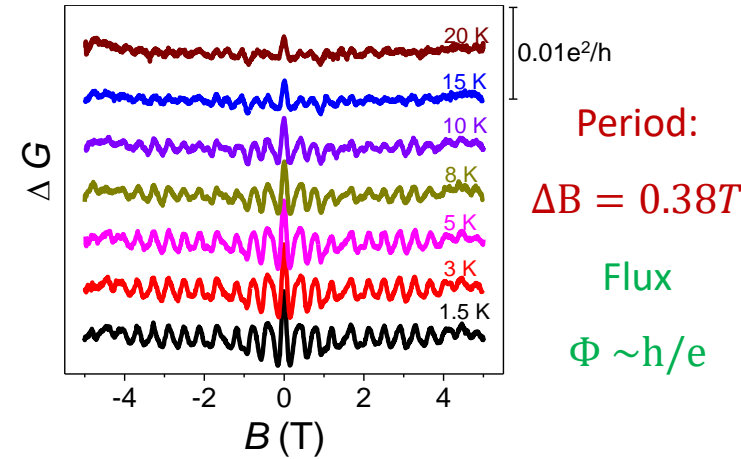
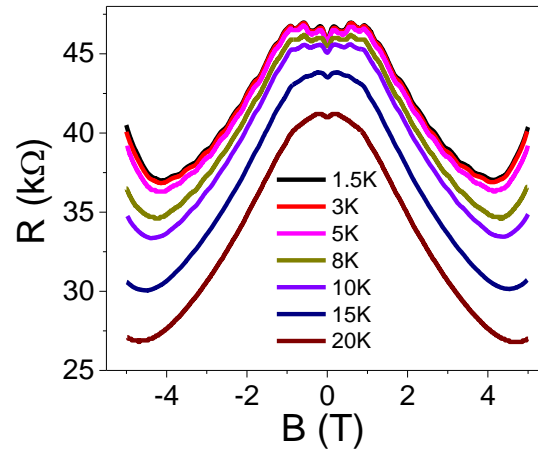
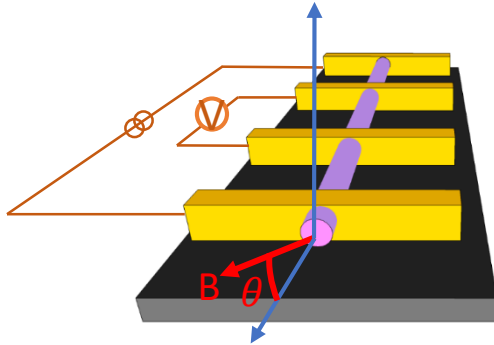
2. Transport properties of Cd_3As_2

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Cd_3As_2 device

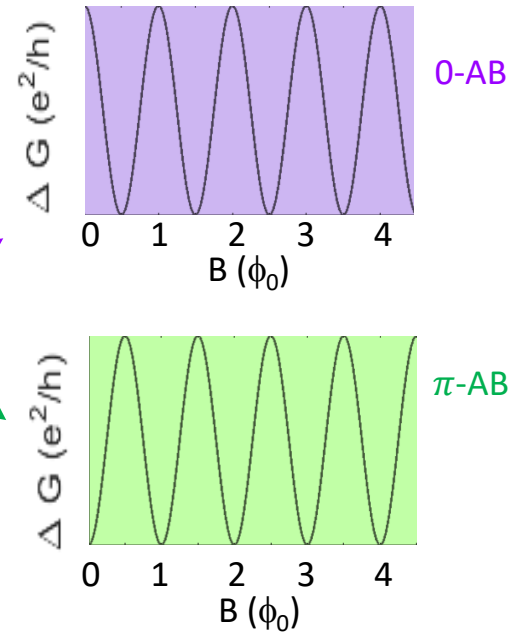
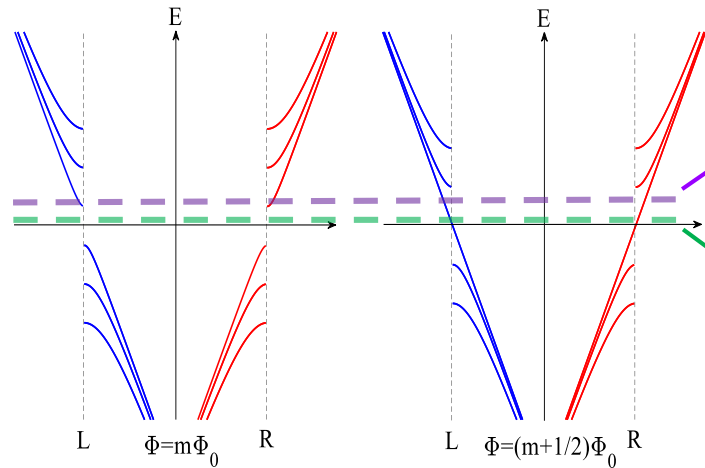
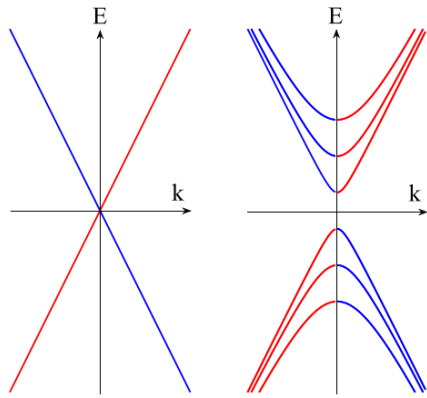


Aharonov-Bohm oscillations



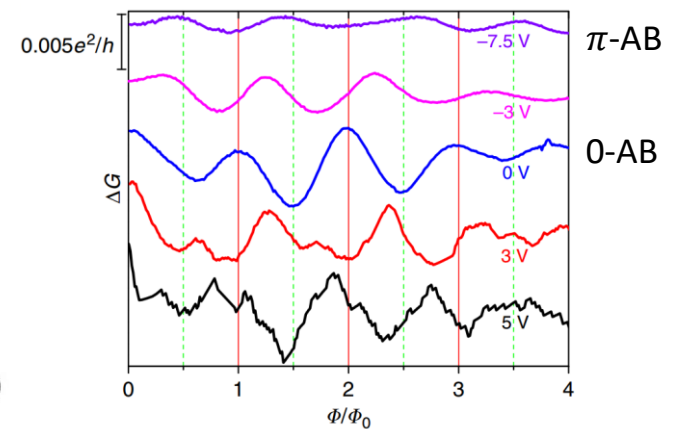
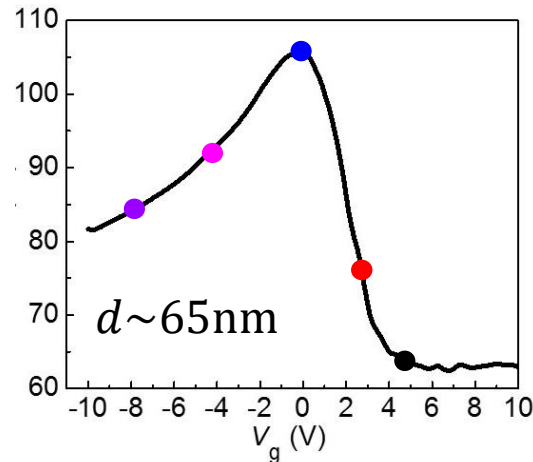
Aharonov-Bohm oscillations

Surface-band splitting due to **quantum confinement**:



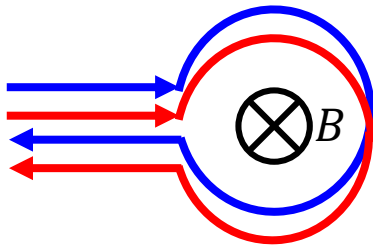
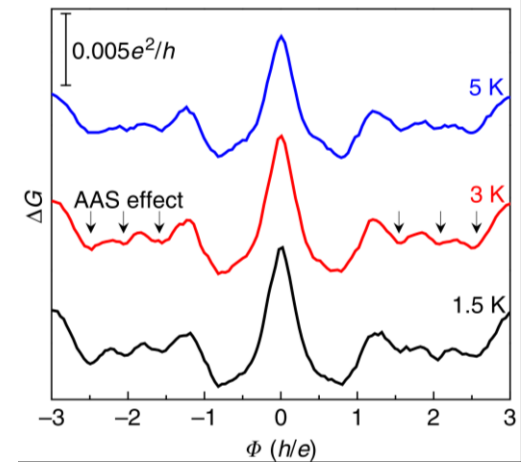
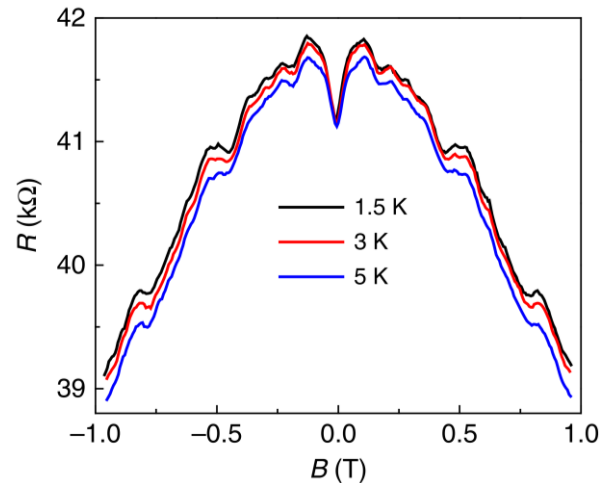
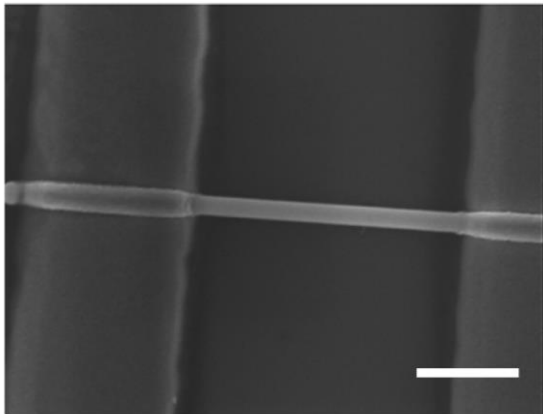
$$E = \pm \hbar v_f \sqrt{\frac{k_{\parallel}^2}{4\pi^2} + \left(\frac{m + \frac{1}{2} - \frac{\Phi}{\Phi_0}}{C} \right)^2}$$

in which, C is perimeter, $m = 0, \pm 1, \pm 2, \dots$



Transport Properties of Cd_3As_2

Altshuler-Aronov-Spivak effect:

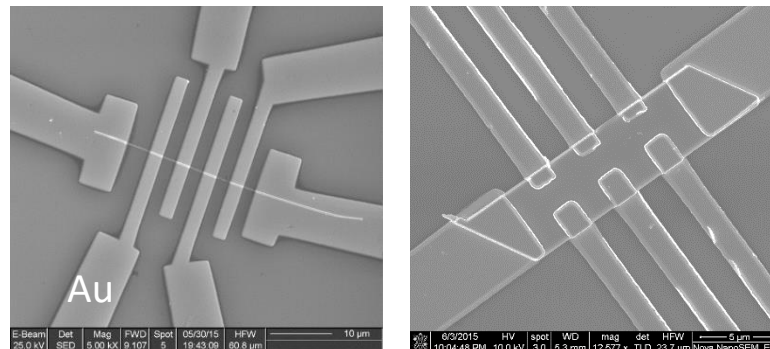


- ✓ diameter: ~ 200 nm
- ✓ AAS oscillations: $h/2e$
- ✓ diffusive transport around the perimeter

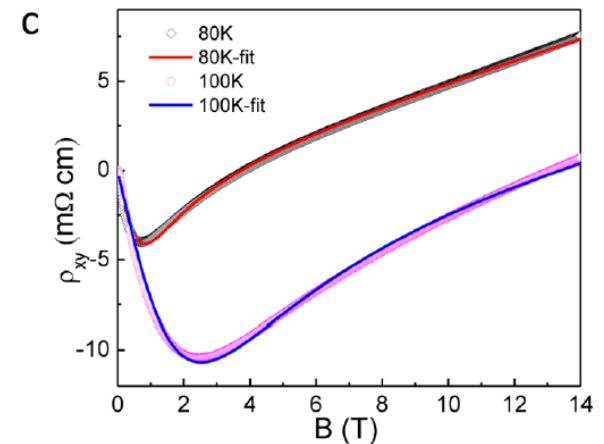
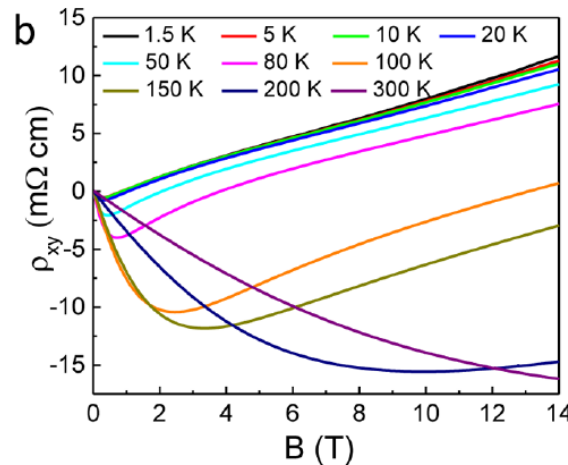
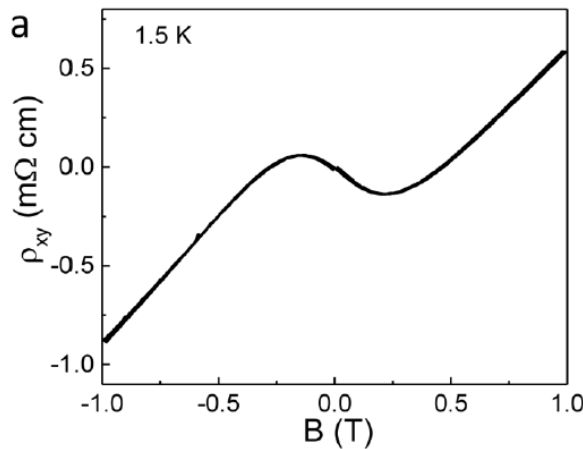
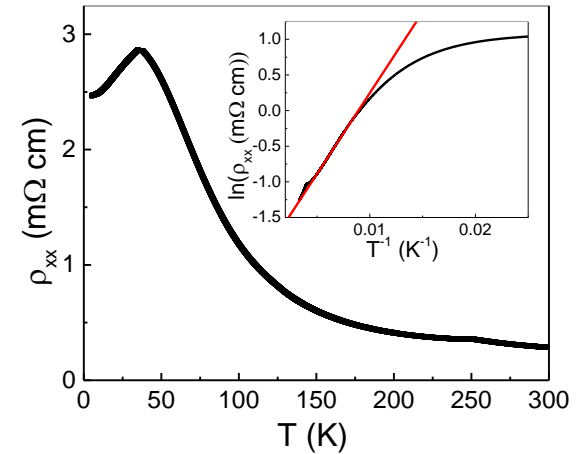
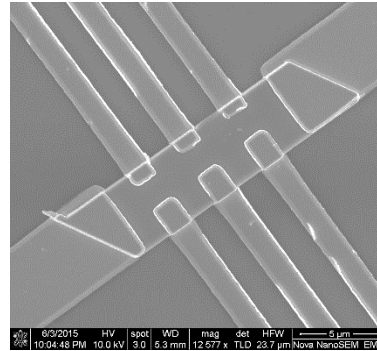
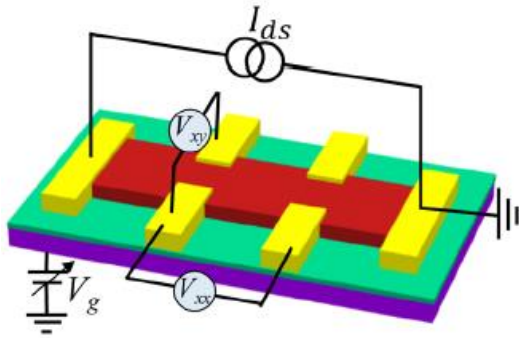
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Cd_3As_2 device



Two carrier transport in Cd_3As_2 nanoplates



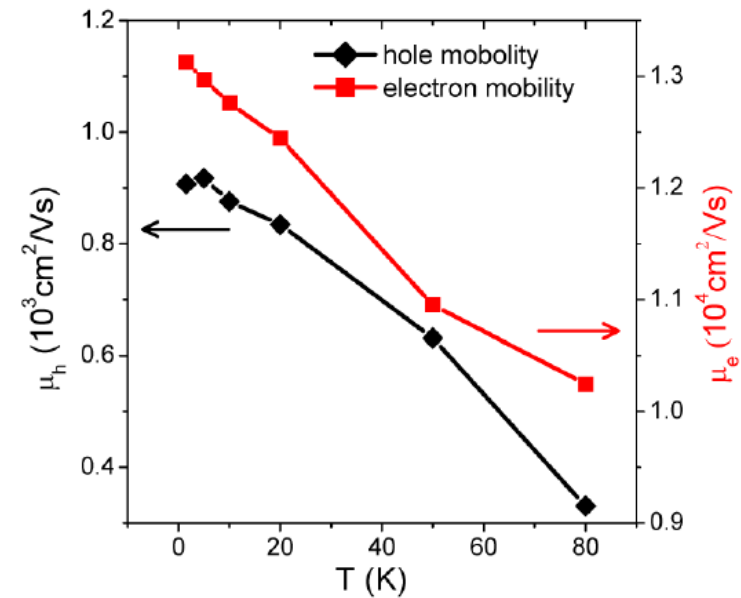
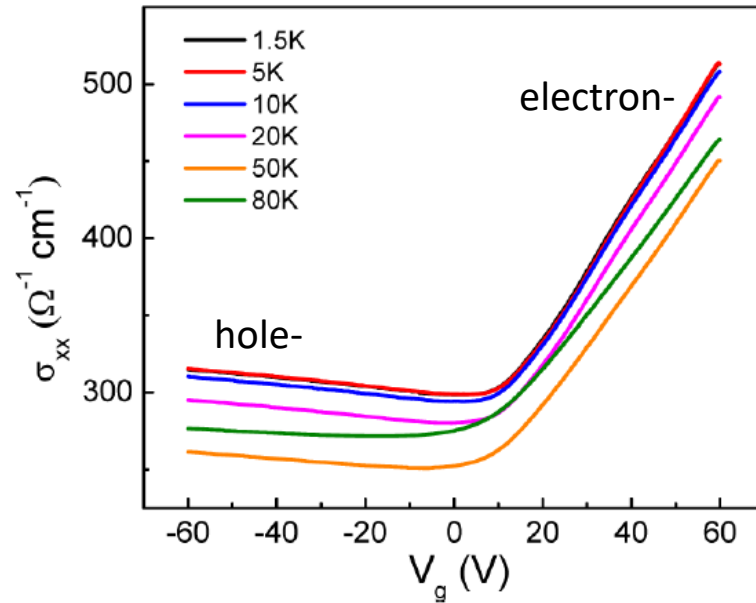
Two-carrier model:

$$\rho_{xy} = \frac{1}{e} \frac{(n_h \mu_h^2 - n_e \mu_e^2) + \mu_h^2 \mu_e^2 B^2 (n_h - n_e)}{(n_h \mu_h + n_e \mu_e)^2 + \mu_h^2 \mu_e^2 B^2 (n_h - n_e)^2} B$$

$n \sim 10^{17} / \text{cm}^3$

Two carrier transport in Cd_3As_2 nanoplates

Gate tunable two-carrier transport:

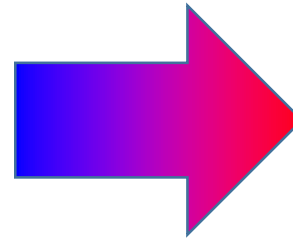
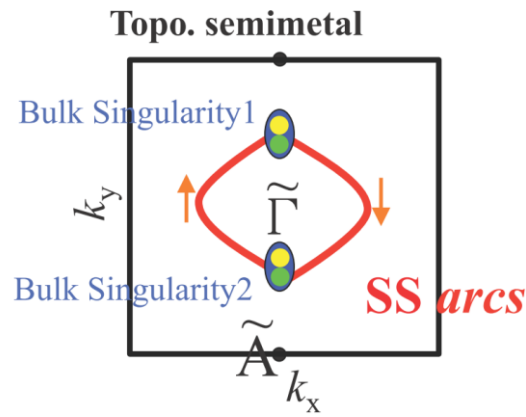
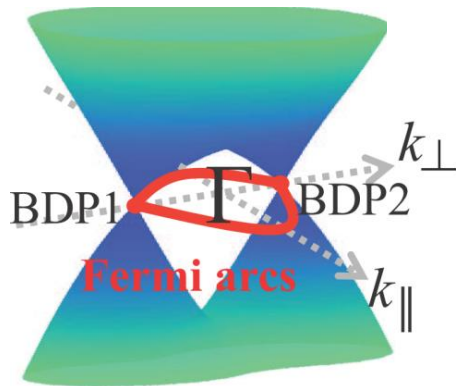
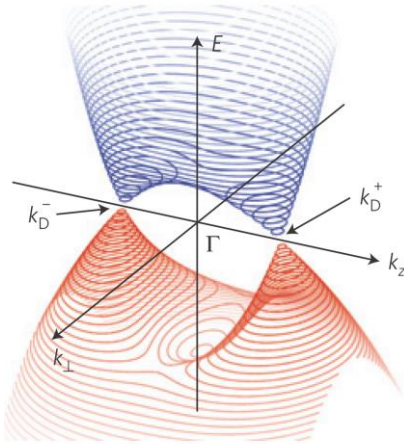


mobility:

$$\mu_e \sim 1 \times 10^4 \text{cm}^2/\text{Vs}$$

$$\mu_h \sim (0.4 - 0.9) \times 10^3 \text{cm}^2/\text{Vs}$$

Research contents



Topological physics of
Dirac fermions and
Fermi arc surface
states

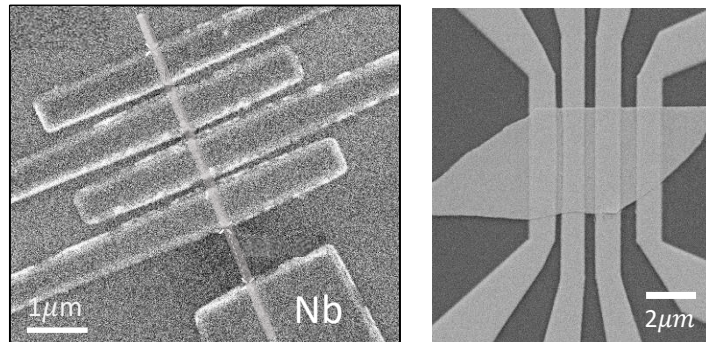
Low-power
electronic devices

Topological
superconductivity

3. Transport properties in Cd_3As_2 based Josephson junctions

- I. Gate-tunable topological superconductivity and 4π -periodic supercurrent (in Cd_2As_2 nanowires)
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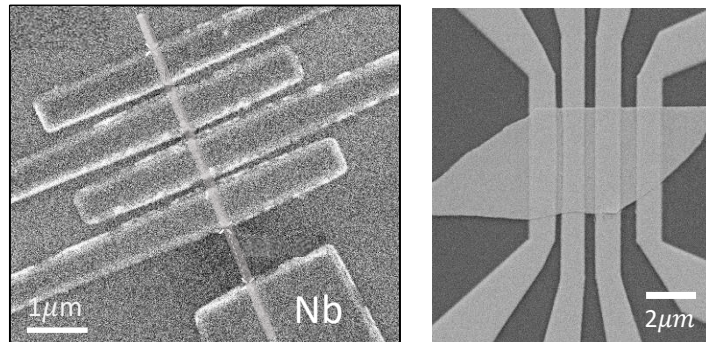
Nb- Cd_3As_2 -Nb Josephson junctions



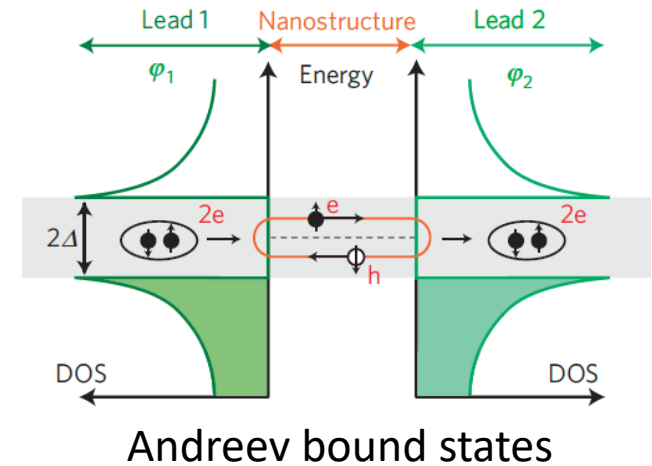
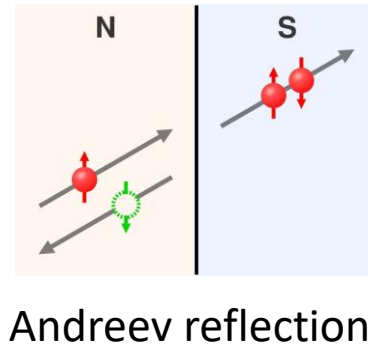
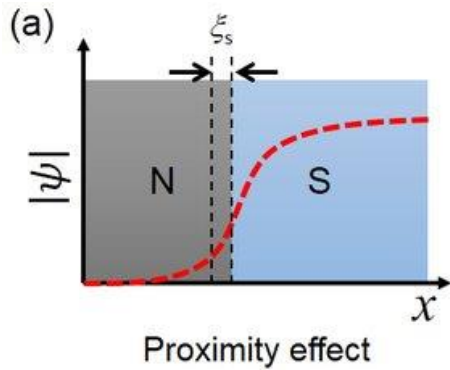
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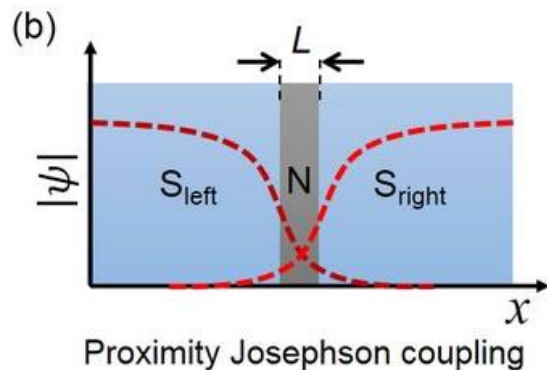
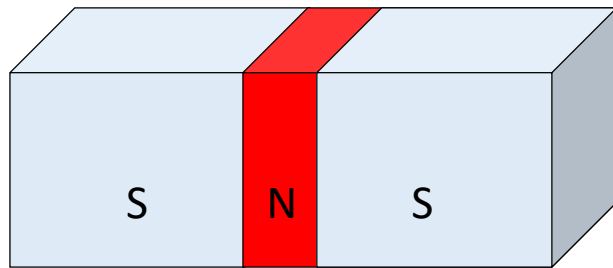
Nb- Cd_3As_2 -Nb Josephson junctions



Josephson junctions



Josephson junction



Order parameter: $\Delta_{1,2} = |\Delta_{1,2}| e^{i\varphi_{1,2}}$

Phase difference: $\varphi = \varphi_1 - \varphi_2$

$I_s = I_c \sin(\varphi)$ d.c. Josephson effect

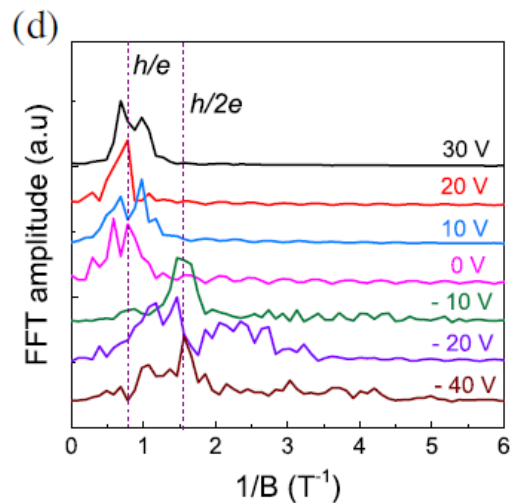
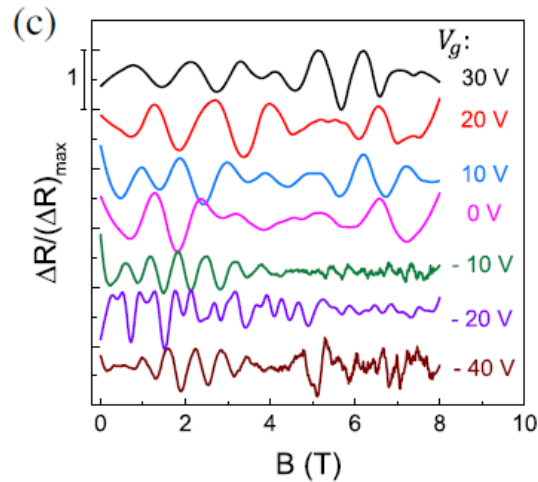
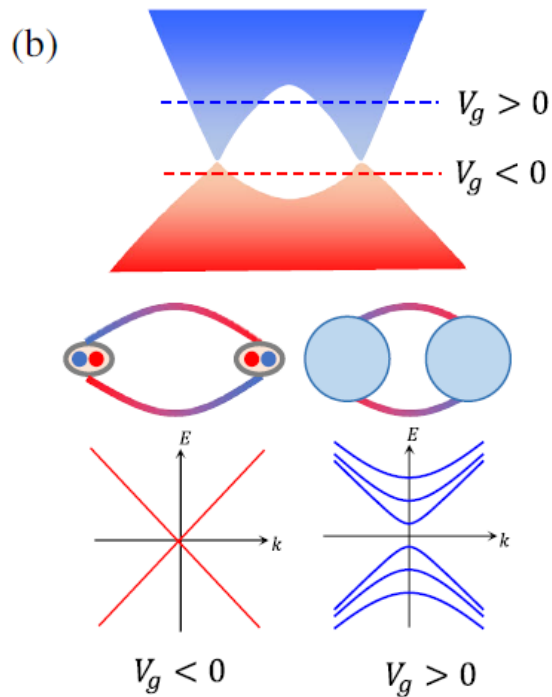
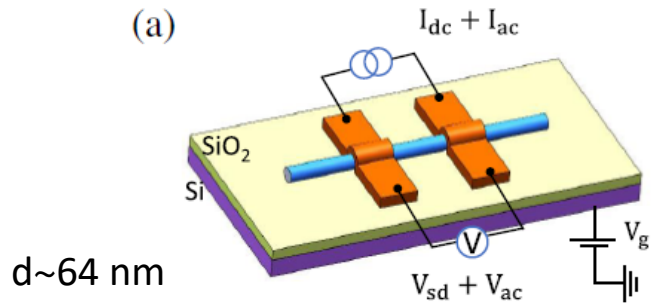
$\frac{\partial \varphi}{\partial t} = \frac{2eV}{\hbar}$ a.c. Josephson effect

Under magnetic field:

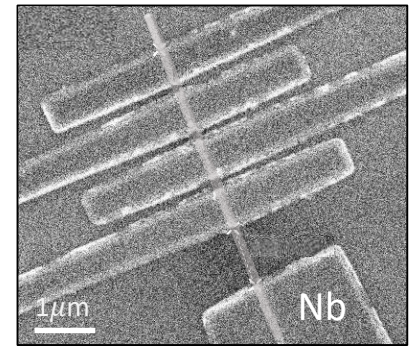
$$I_c(B) = I_{c0} \left| \frac{\sin(\frac{\pi \Phi}{\Phi_0})}{\frac{\pi \Phi}{\Phi_0}} \right|$$

Gate control of AB and AAS interference

Normal state transport at $T=10$ K



Nb-Cd₃As₂ nanowire-Nb

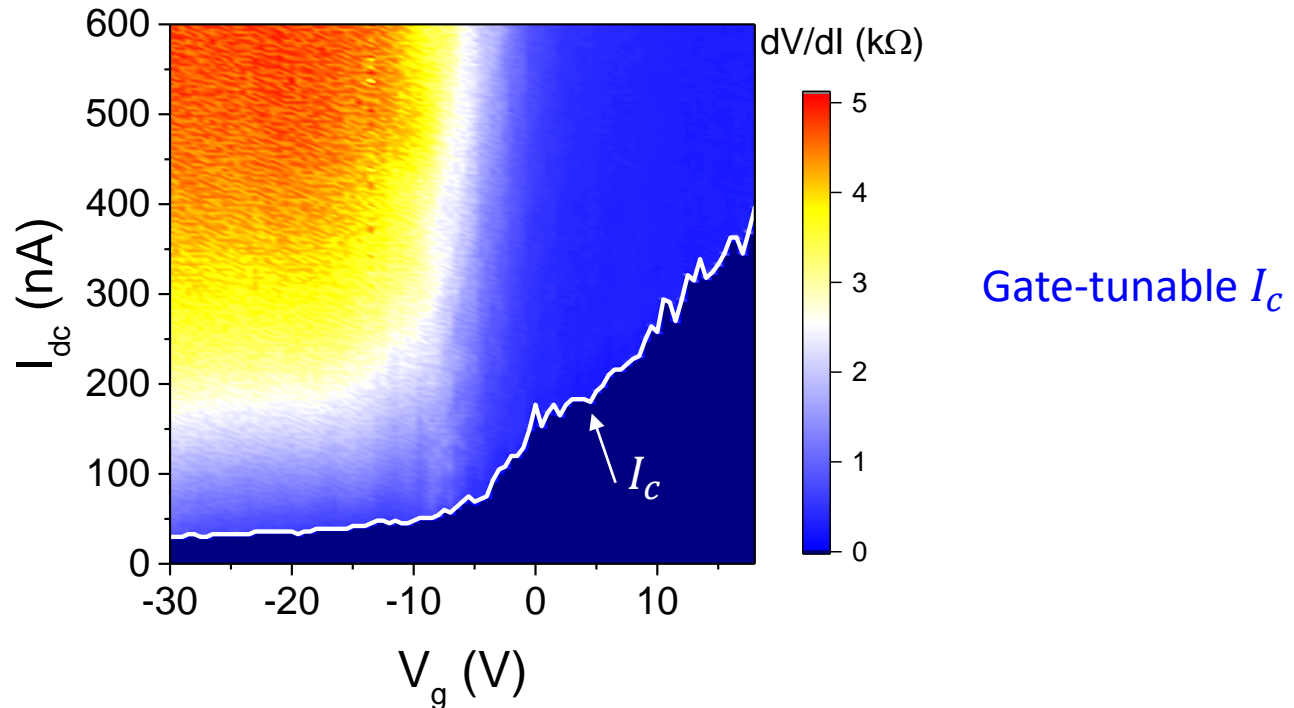
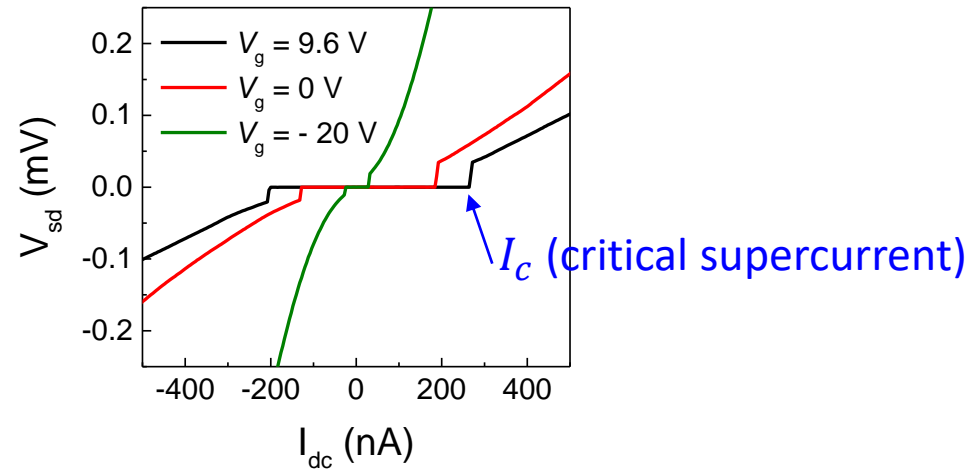
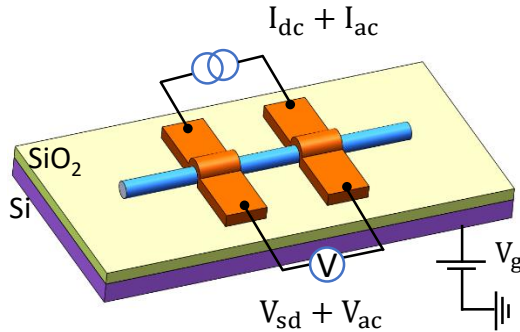


AB oscillations: h/e

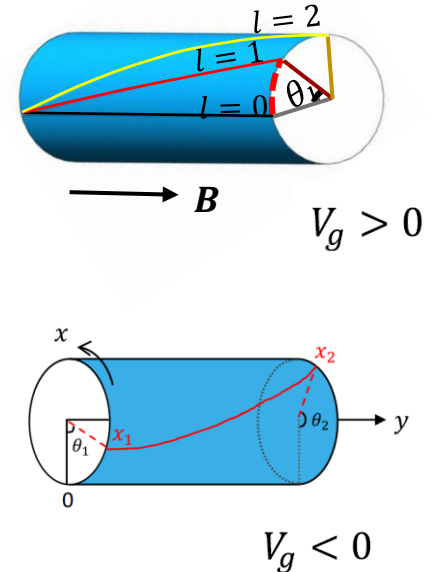
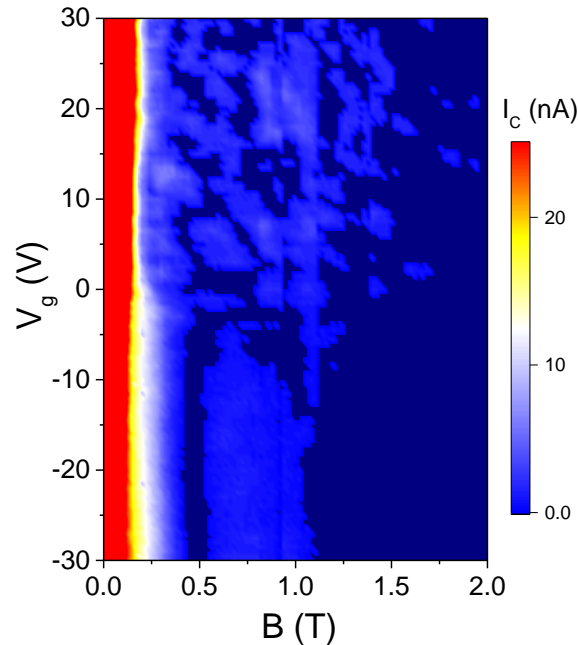
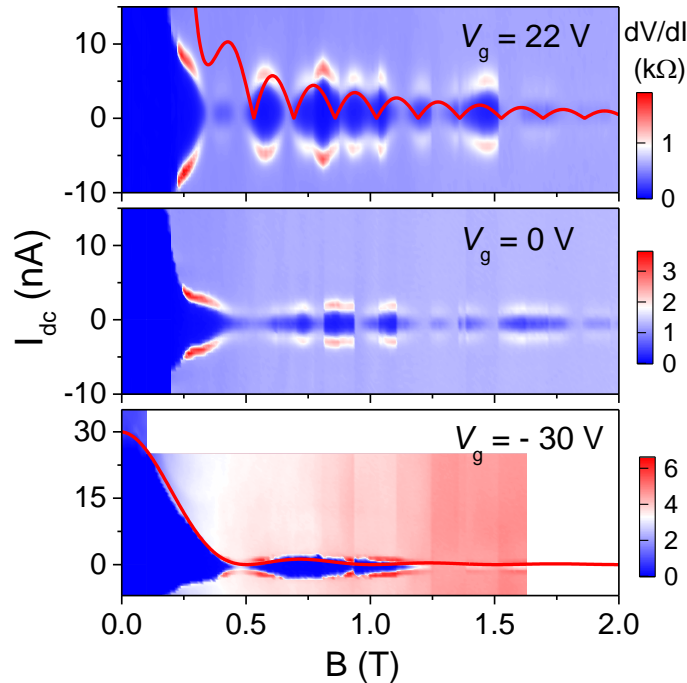
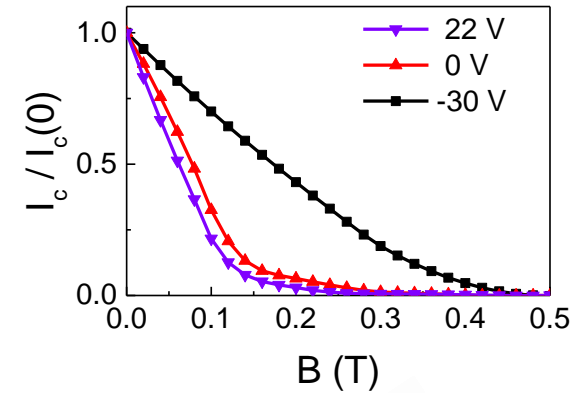
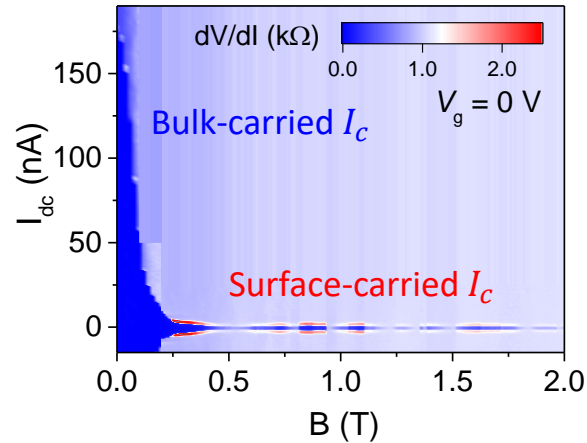
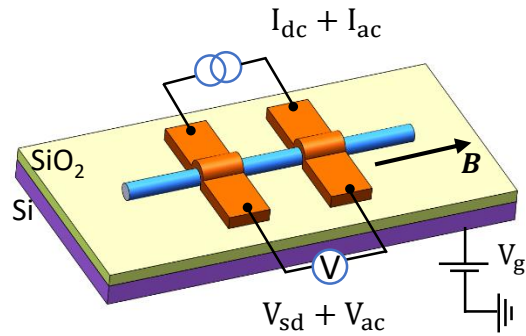
AAS oscillations: $h/2e$

Proximity-induced supercurrent in Cd_3As_2 nanowires

$T = 10 \text{ mK}$

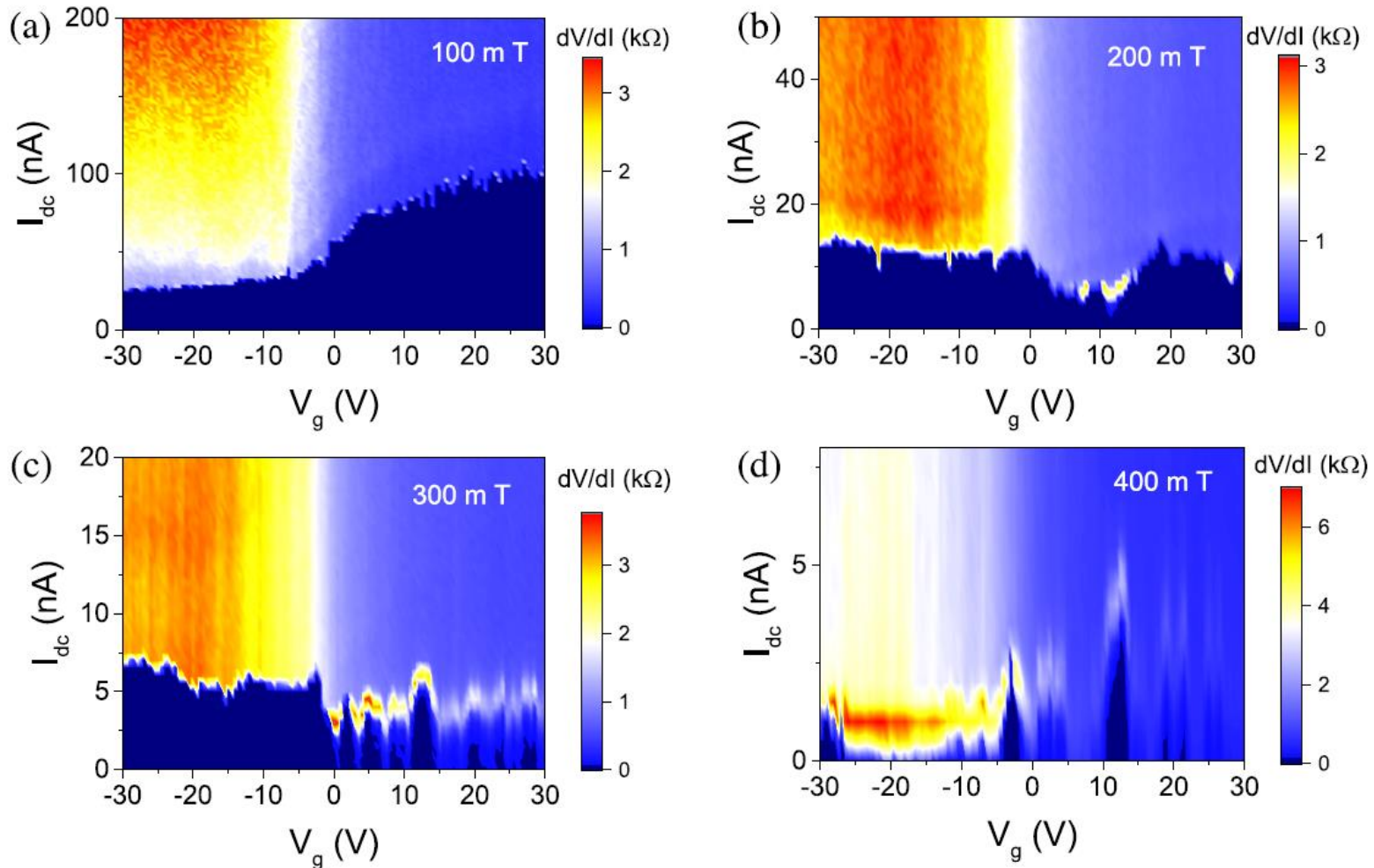


Quantized orbit interference induced supercurrent oscillations

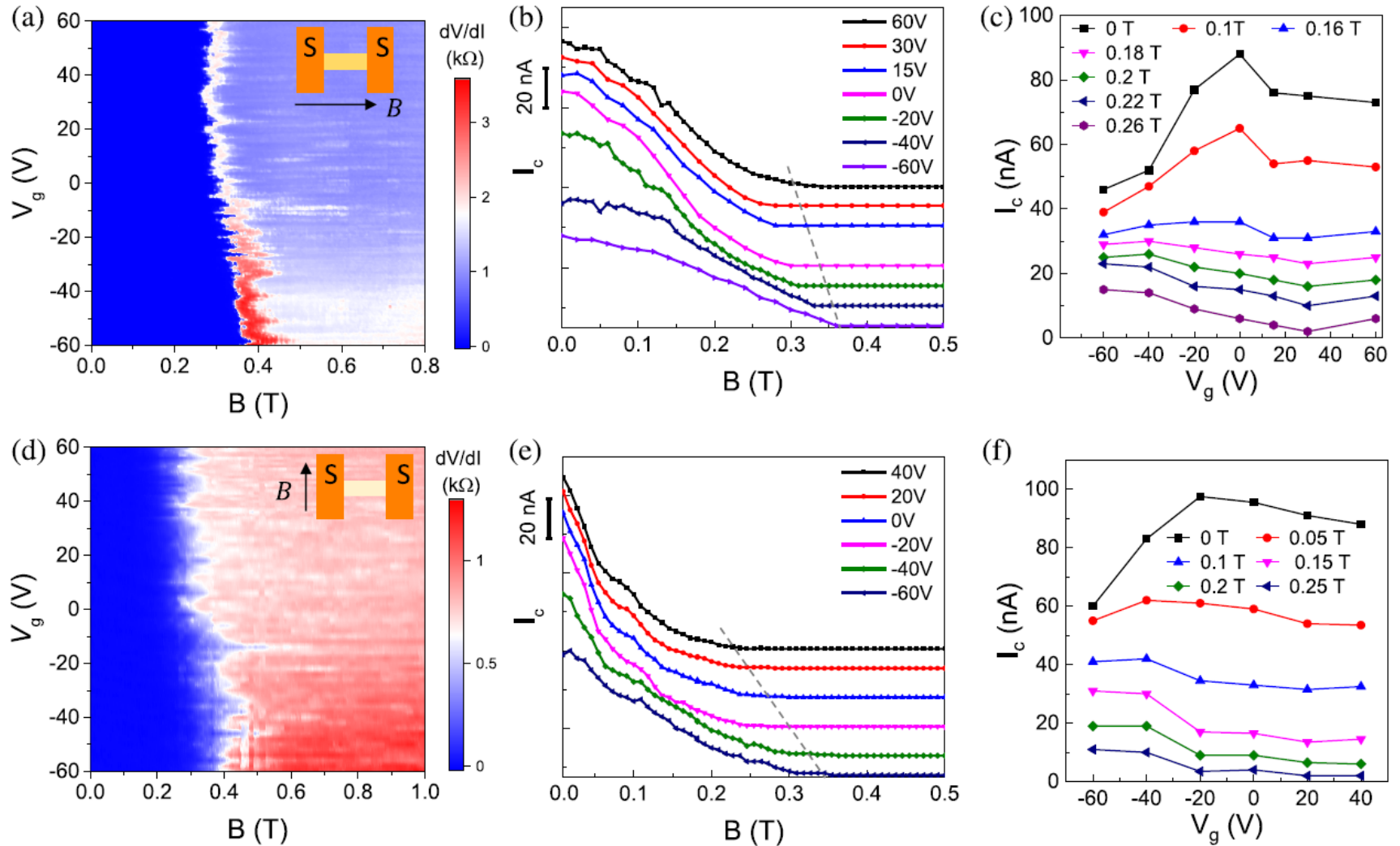


Gate control of topological phase transition

Applying a magnetic field to suppress the bulk-carried supercurrent



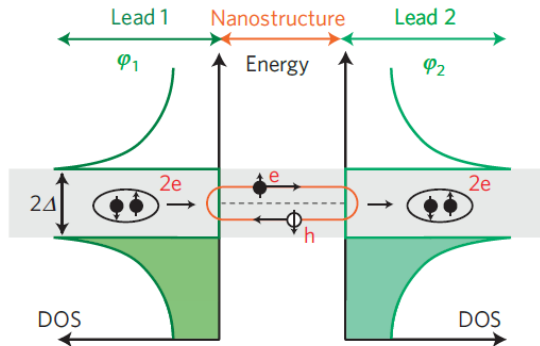
Topological protection enhance critical magnetic field B_c



RF irradiation and 2π -periodic supercurrent

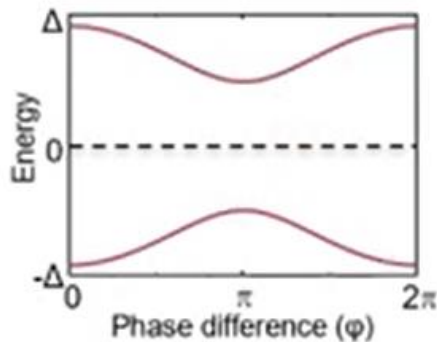
- Conventional proximity effect

Andreev bound states



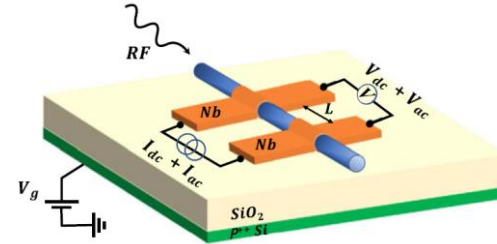
Nat. Phys. **6**, 965–969 (2010)

- 2π -periodic spectrum



$$I = \frac{2e}{\hbar} \frac{\partial E}{\partial \varphi} = I_0 \sin \varphi$$

2π -periodic



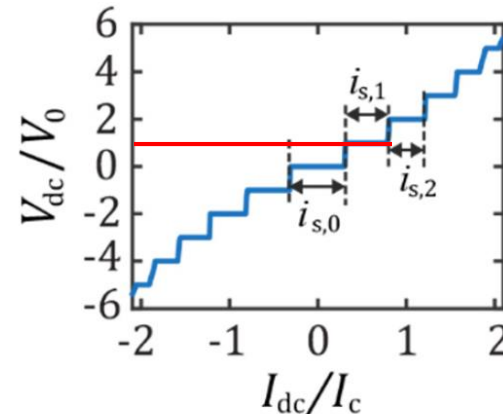
- Inverse a.c. Josephson effect

$$V(t) = V + V_0 \cos(\omega' t + \theta)$$

$$I_s = I_c \sum_n (-1)^n J_n \frac{2eV_0}{\hbar\omega'} \sin[(\omega - n\omega') - n\theta + \varphi_0]$$

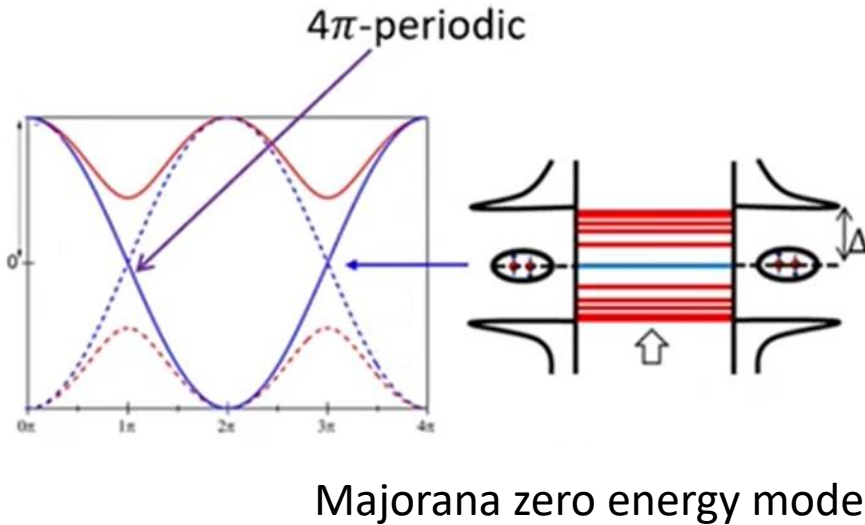
当 $\omega = n\omega'$ 时: $I_s = (-1)^n I_c J_n \frac{2eV_0}{\hbar\omega'} \sin(\varphi_0 - n\theta)$

DC components: $V_{dc} = n \frac{\hbar f}{2e}$ (Shapiro steps) ($\omega' = 2\pi f$)



Phys. Rev. B **103**, 235428 (2021)

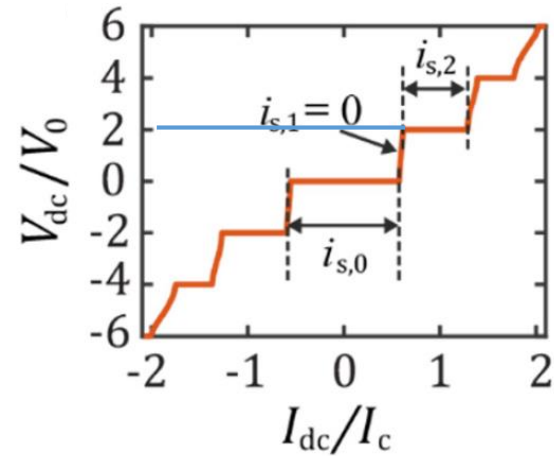
4π -periodic supercurrent and even-integer Shapiro steps



Majorana = "half state"

$$I_{MBS} = \frac{2e}{\hbar} \frac{\partial E}{\partial \varphi} = I_0 \sin \frac{\varphi}{2}$$

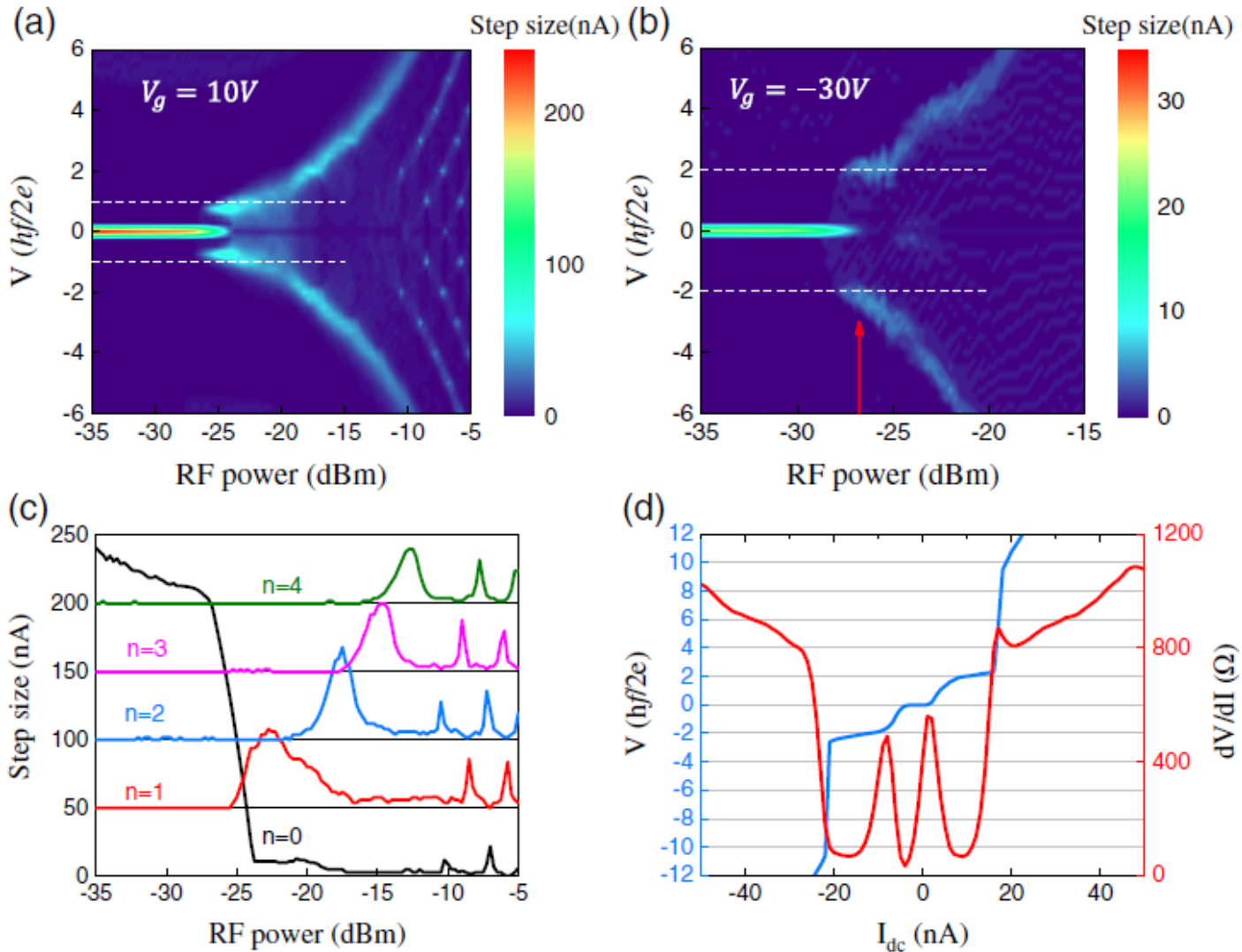
4 π -periodic



$$V_{dc} = \boxed{2n} \frac{\hbar f}{2e}$$

Shapiro step at even-integer

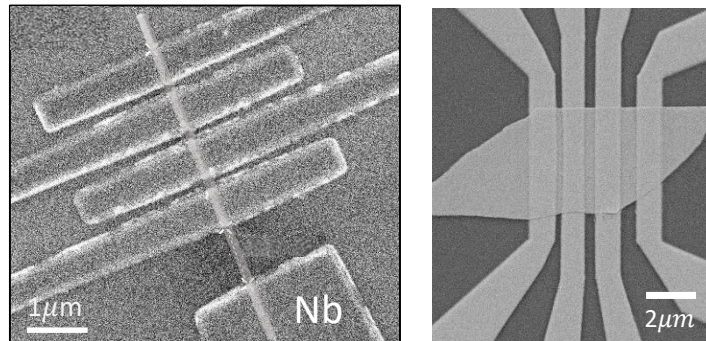
4π -periodic supercurrent from surface states



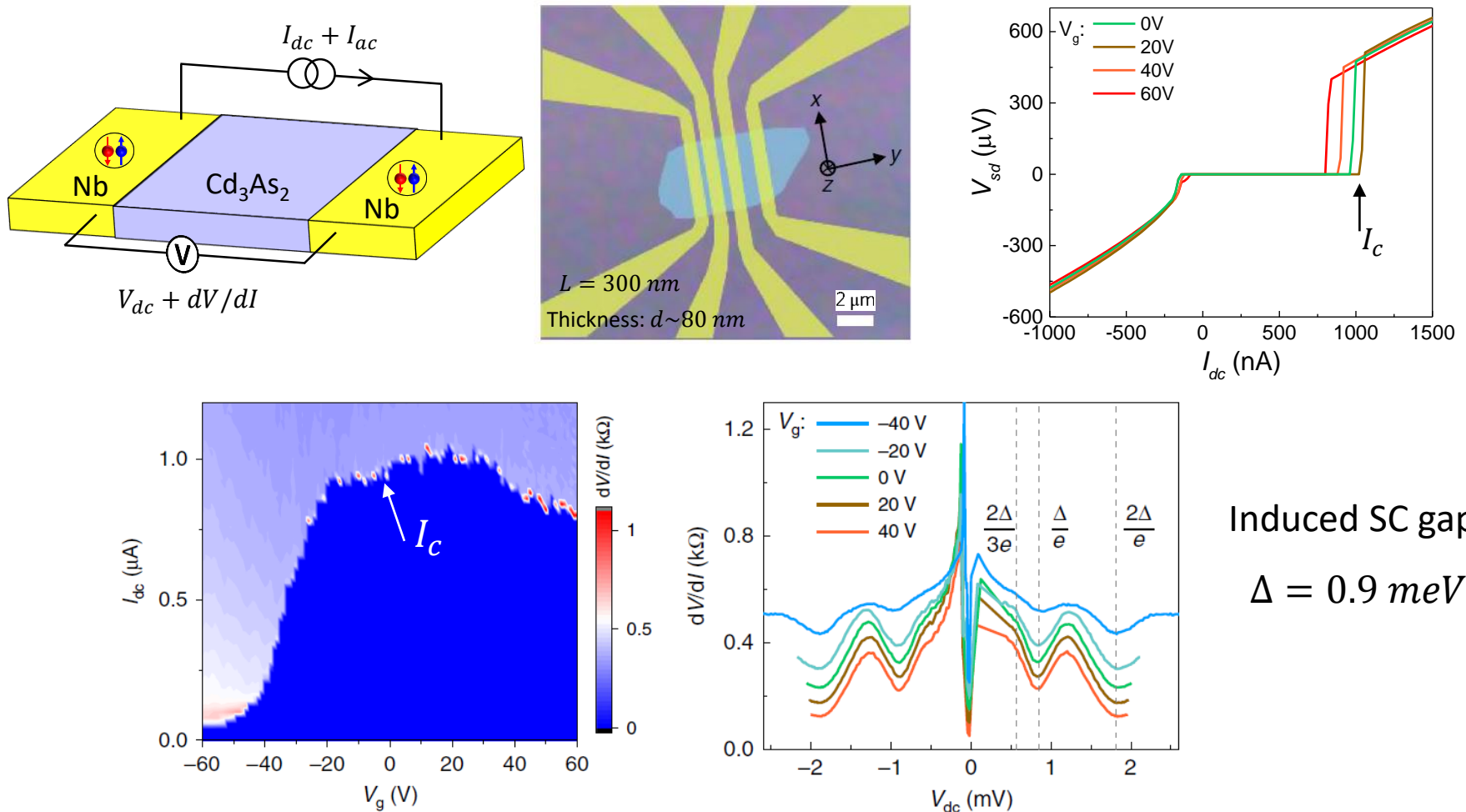
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Nb- Cd_3As_2 -Nb Josephson junctions



Nb-Cd₃As₂-Nb Josephson junction

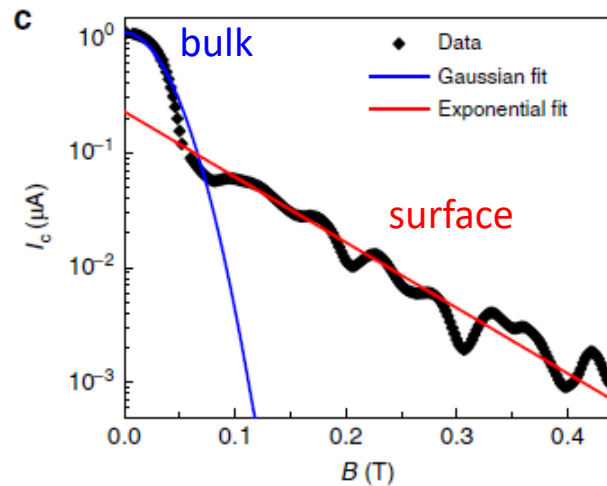
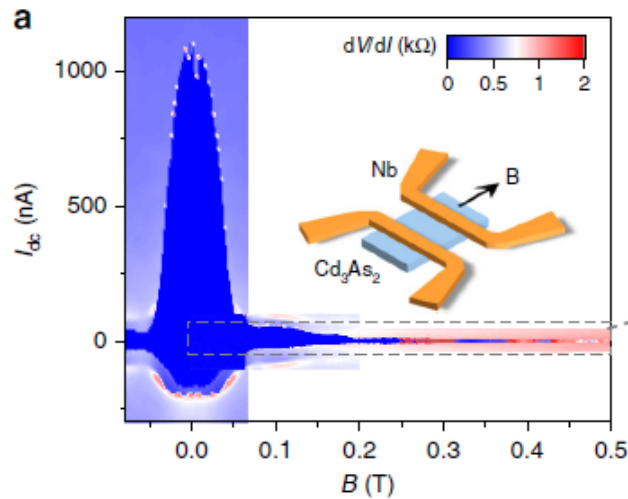


- Gate-tunable supercurrent
- Multiple Andreev Reflection (MAR)

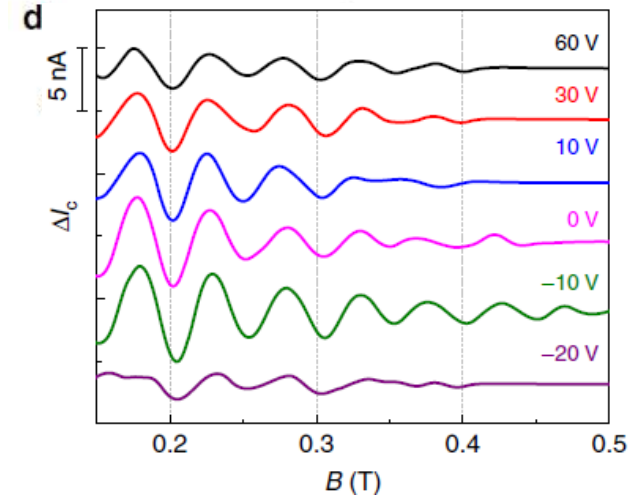
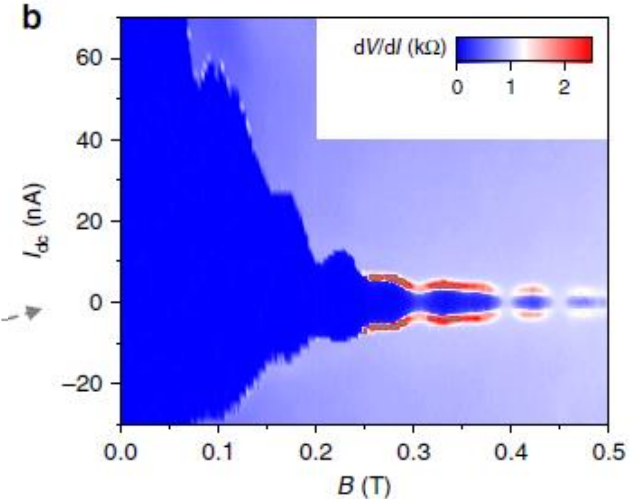
Induced SC gap:
 $\Delta = 0.9 \text{ meV}$

Supercurrent oscillations under parallel B

Two-channel transport

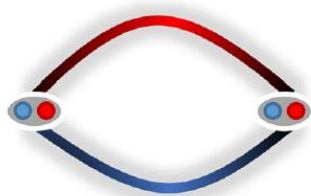
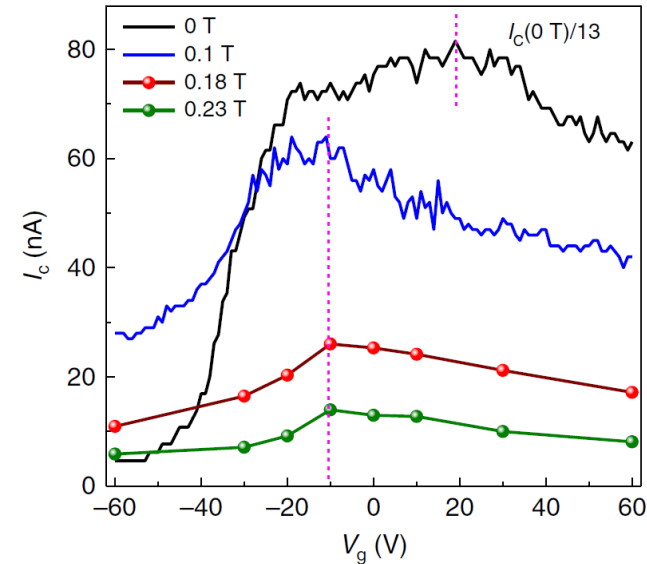
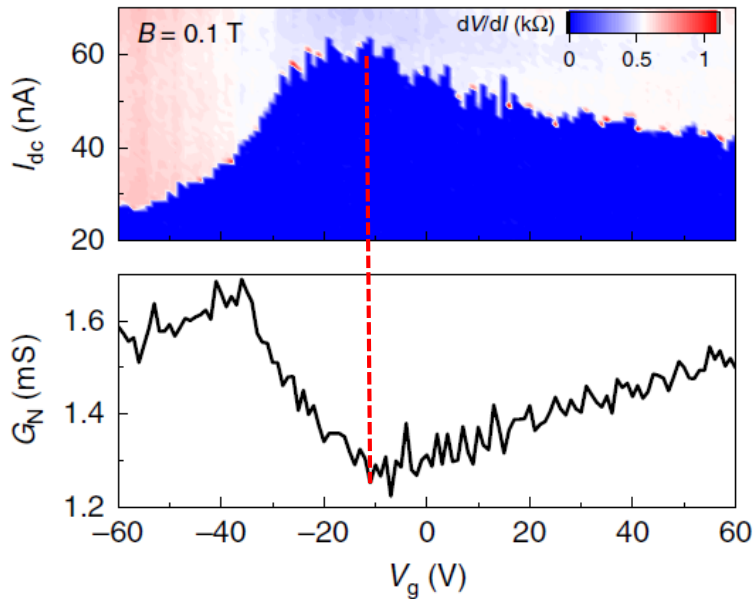


I_c oscillations at higher fields



Two-channel transport: bulk and surface

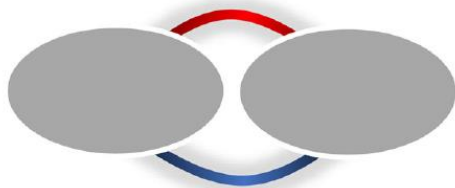
Applying a magnetic field (0.1T) to suppress the bulk-carried supercurrent



Near Dirac point



At $V_g = -10$ V:
 I_c , maximum
 G_N , minimum



Far from Dirac point

Supercurrent oscillations: orbital effect

- Zeeman effect: finite momentum pairing (excluded)
- magnetic misalignment (excluded)
- spin-orbital phase shift (excluded)
- **orbital effect**

$$\text{orbital phase: } \frac{2\pi}{\Phi_0} \int_{x_1}^{x_2} \mathbf{A} \cdot d\mathbf{l}$$

phase difference between x_1 and x_2 :

$$\phi_1(x_1) - \phi_2(x_2) = \frac{\pi \mathbf{B}(x_1 - x_2)t}{\Phi_0}$$

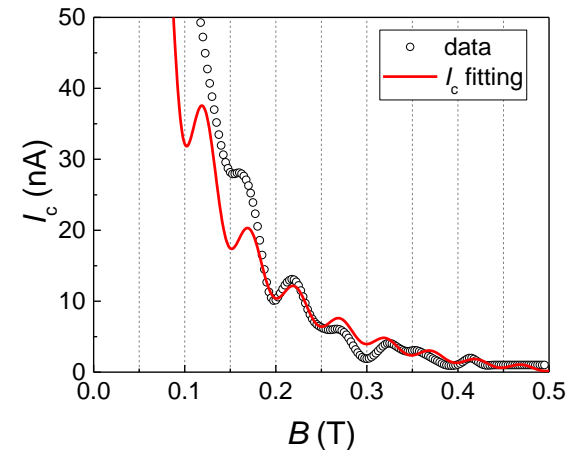
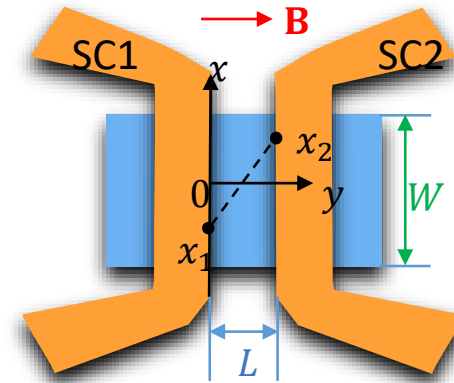
$$I_{\text{surface}}(\Delta\phi, \mathbf{B})$$

$$= \int_{-\frac{W}{2}}^{\frac{W}{2}} \int_{-\frac{W}{2}}^{\frac{W}{2}} dx_1 dx_2 \frac{1}{\mathbf{r}^\varepsilon} \sin(\Delta\phi + \phi_1(x_1) - \phi_2(x_2))$$

where $\mathbf{r} = \sqrt{L^2 + (x_1 - x_2)^2}$

ε denotes the phase coherent strength along the x direction

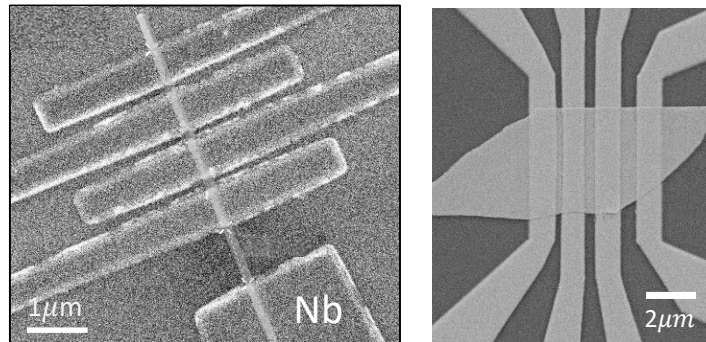
t : nanoplate thickness



3. Transport properties in Cd_3As_2 based Josephson junctions

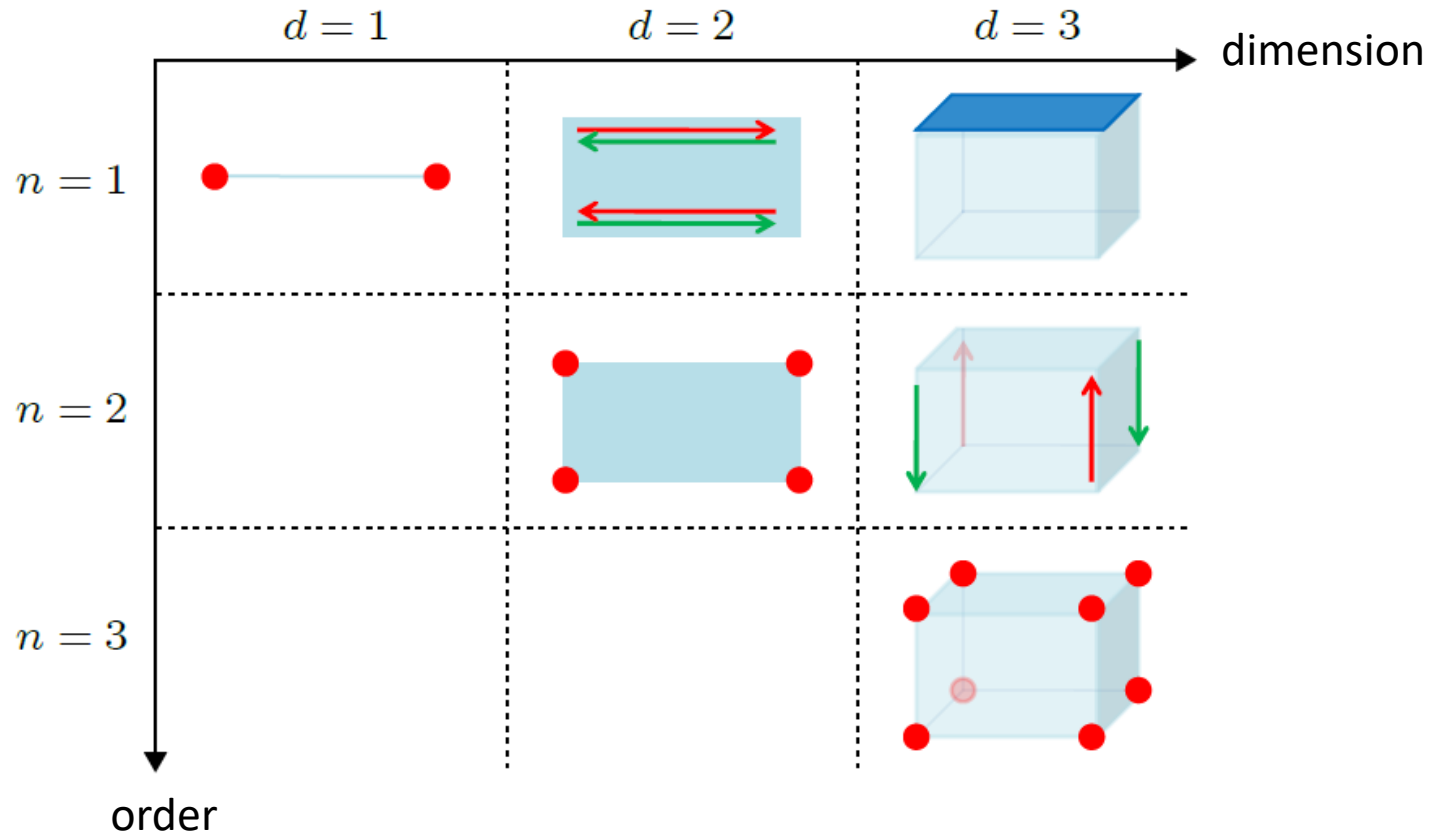
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Nb- Cd_3As_2 -Nb Josephson junctions



Higher order topological states

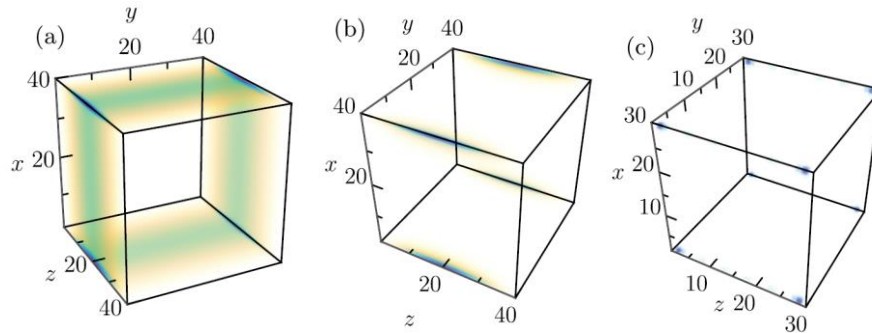
A d -dimensional system n -th order topological phase holds gapless states that live on $(d - n)$ dimensional subsystems.



Cd_3As_2 : a Higher order topological semimetal

➤ Phys. Rev. B 99, 041301(R) (2019) [Bitan Roy *et al.*]

Higher-order topological phases: A general principle of construction

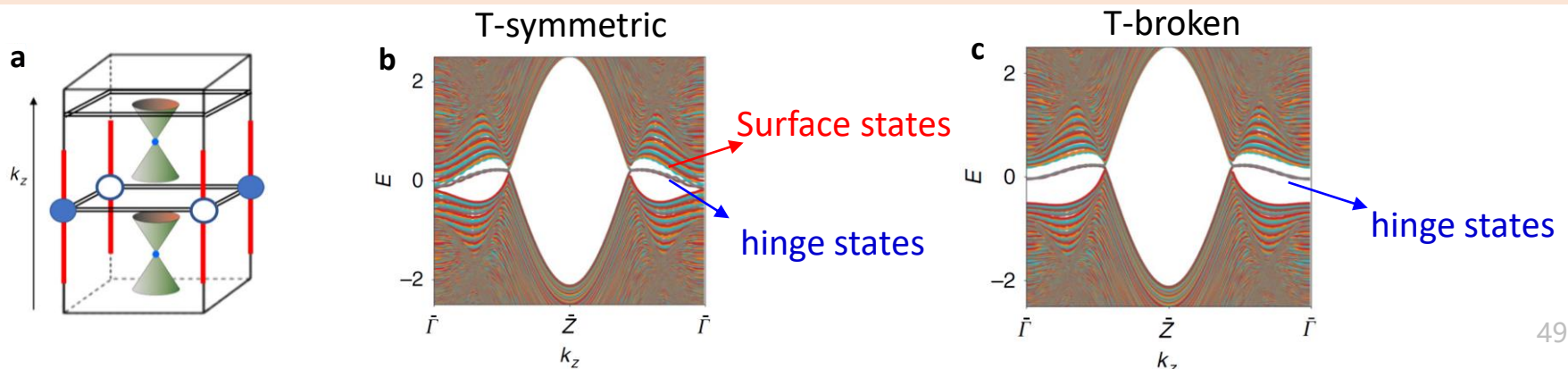


To summarize, we here present a general principle for constructing HOT phases, including gapped [46] and gapless representatives. In particular, we illustrate the higher-order generalization of a topological Kondo insulator (see Fig. 1), TDSMs (see Figs. 2 and 3), which can be realized in Cd_3As_2 [51], Na_3Bi [52], and a NLSM (see Fig. 4), relevant for Ca_3P_2 [53], when subjected to lattice deformations thereby reducing the symmetry. This construction can also be applied to gapless

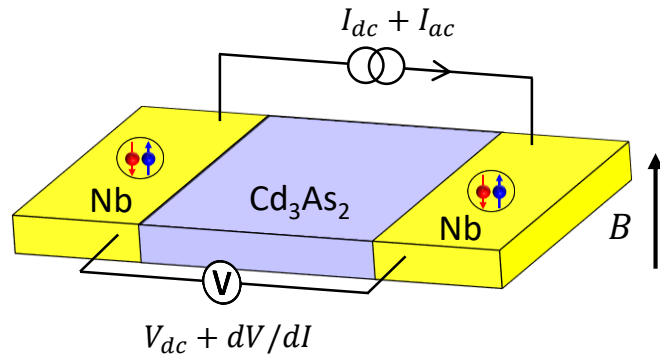
➤ Nat. Commun. 11, 13, 627 (2020). [B. Andrei Bernevig *et al.*]

Strong and fragile topological Dirac semimetals with higher-order Fermi arcs.

- The surface Fermi arcs in Dirac semimetals can be disconnected and removed without breaking a symmetry or closing a gap, therefore are not topological consequences of the bulk Dirac points themselves.
- The surface states appear due to the topology of high symmetry planes, and are not required to connect the surface projections of the bulk Dirac points.

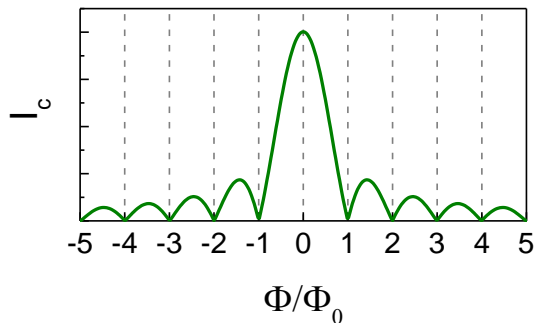
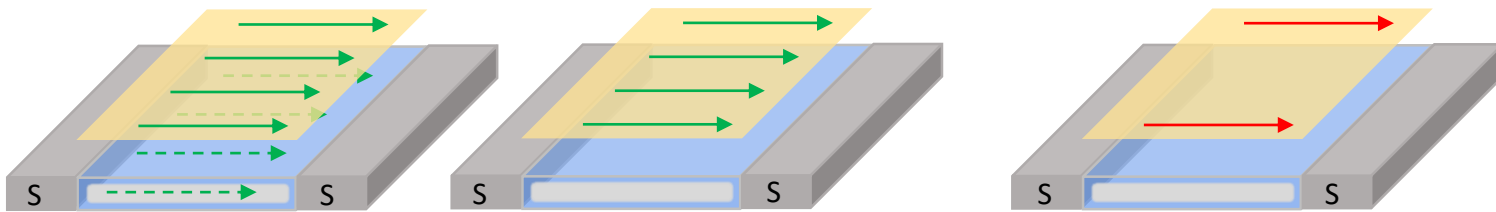


Transport modes filter by supercurrent interference

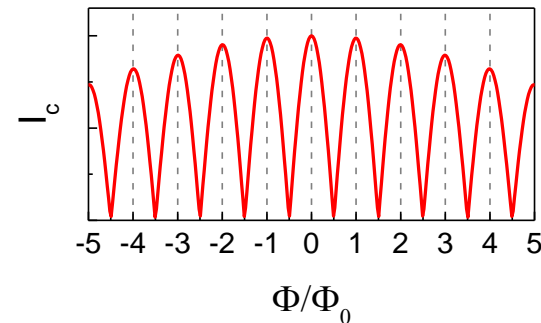


$$I_c(B) = |J_c(B)|$$

$$= \left| \int_{-\infty}^{\infty} J_c(x) \exp(i2\pi L_{eff} B x / \Phi_0) dx \right|$$



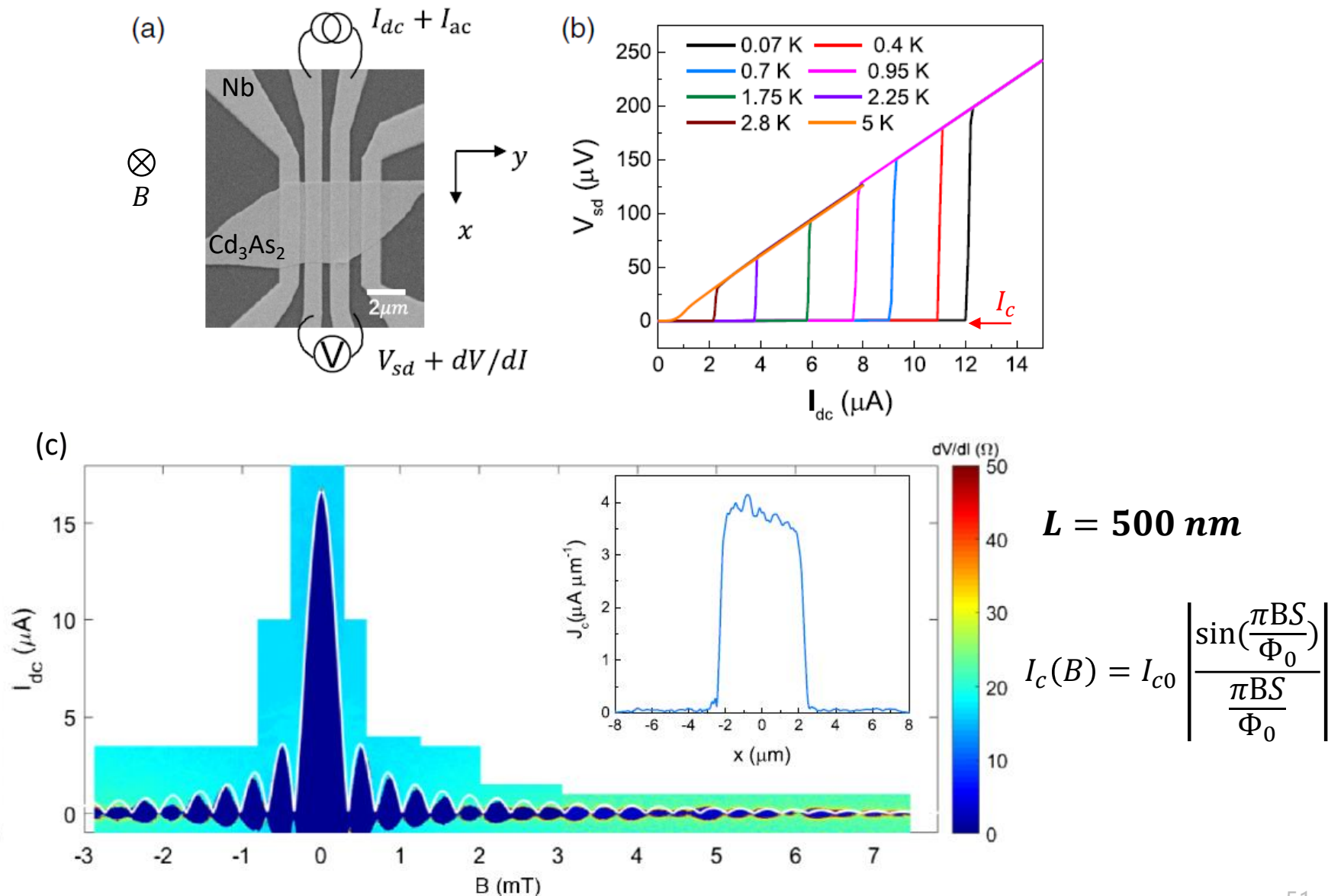
Fraunhofer pattern



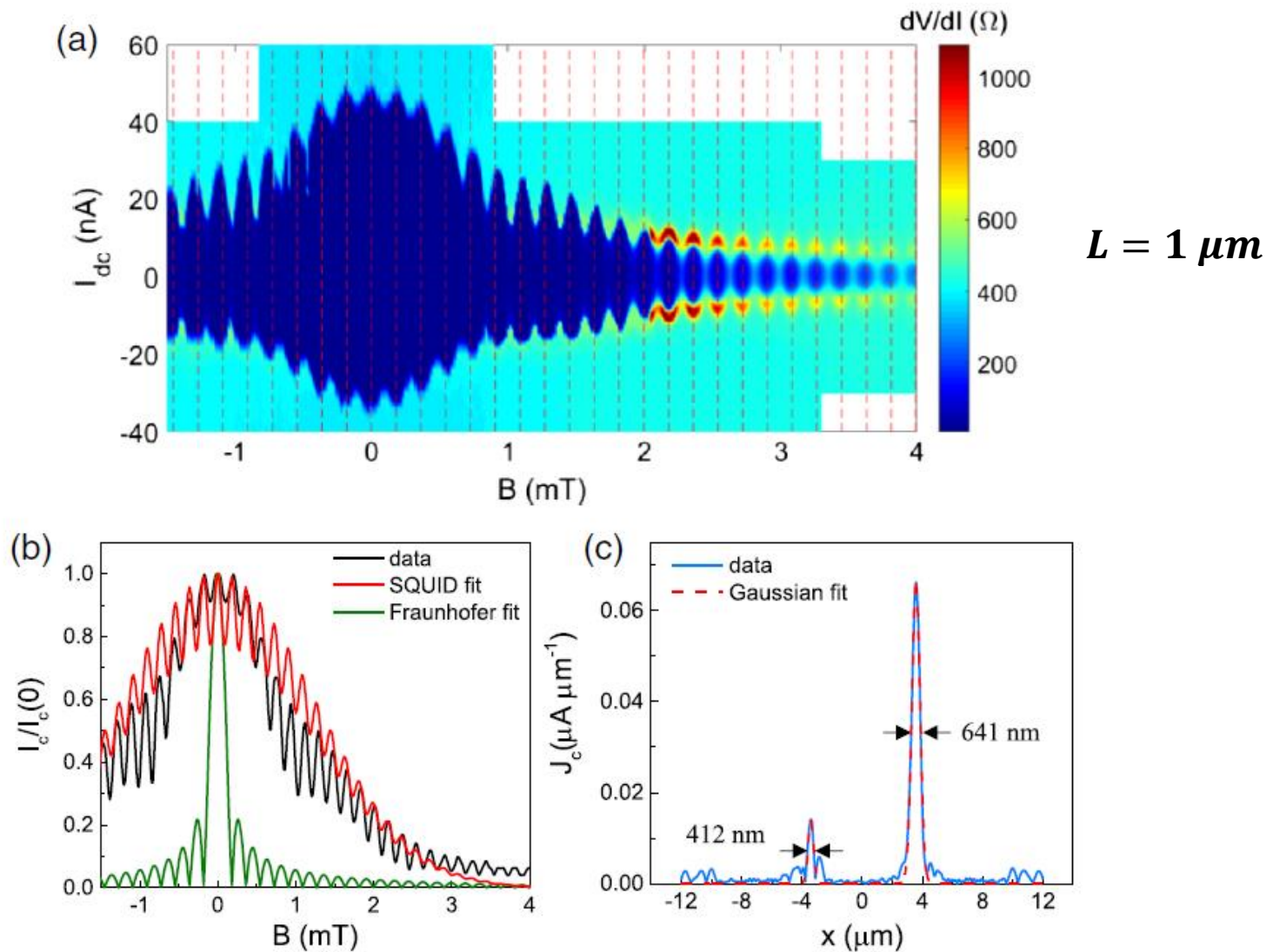
SQUID pattern

(superconducting quantum interference device)

Fraunhofer pattern under perpendicular B



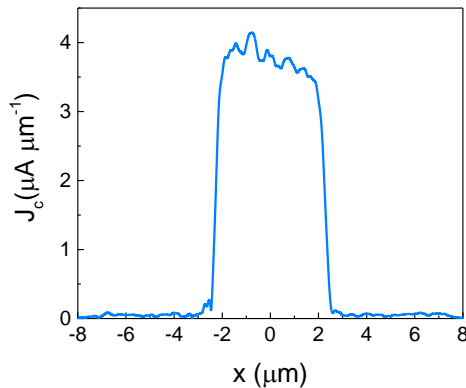
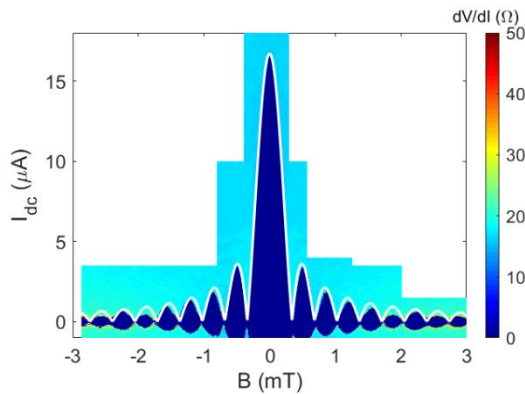
SQUID pattern in a long junction



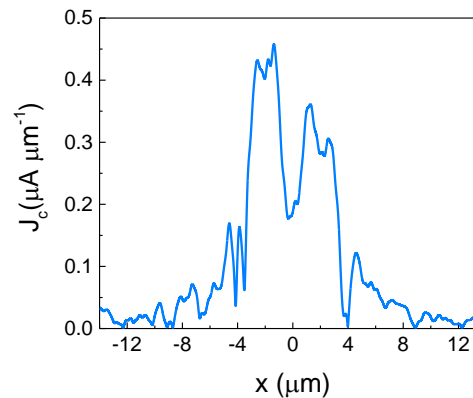
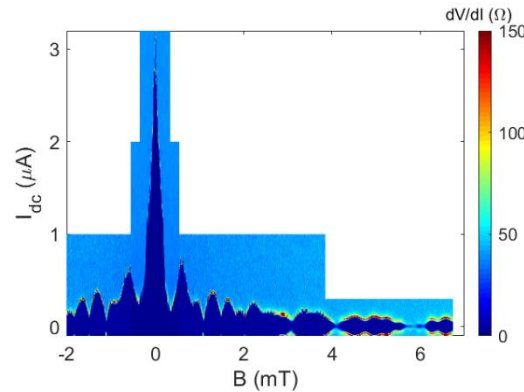
Reducing electronic transport dimension to topological hinge states by increasing device size

Increasing junction length

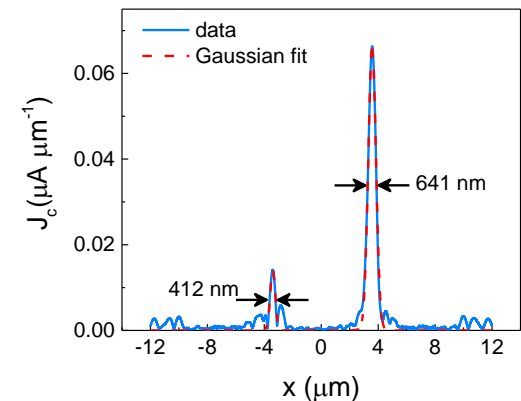
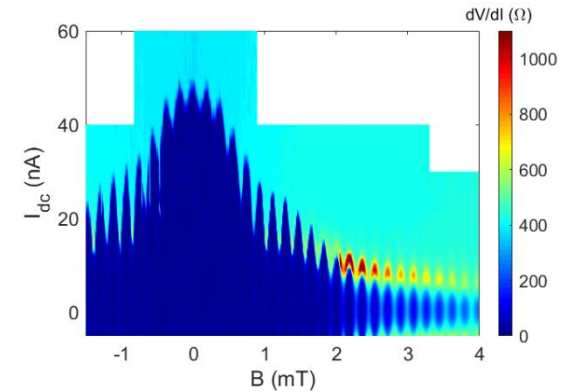
$L = 500 \text{ nm}$, 3D



$L = 800 \text{ nm}$, 2D



$L = 1 \mu m$, 1D



Superconducting coherence length: $l_{\phi}^{3D \text{ bulk}} < l_{\phi}^{2D \text{ surface}} < l_{\phi}^{1D \text{ hinge}}$

Summary

Transport properties of Cd_3As_2 nanostructures

- I. Chiral anomaly effect in Cd_3As_2 nanowires and nanoplates
- II. Aharonov-Bohm oscillations in Cd_3As_2 nanowires
- III. Two carrier transport in Cd_3As_2 nanoplates

Transport properties of Nb- Cd_3As_2 -Nb Josephson junctions

- I. Gate-tunable topological superconductivity and 4π -periodic supercurrent (in Cd_2As_2 nanowires)
- II. Fermi-arc supercurrent oscillations (in Cd_3As_2 nanoplates)
- III. Higher order topological hinge states (in Cd_3As_2 nanoplates)



Thank you for your attention!